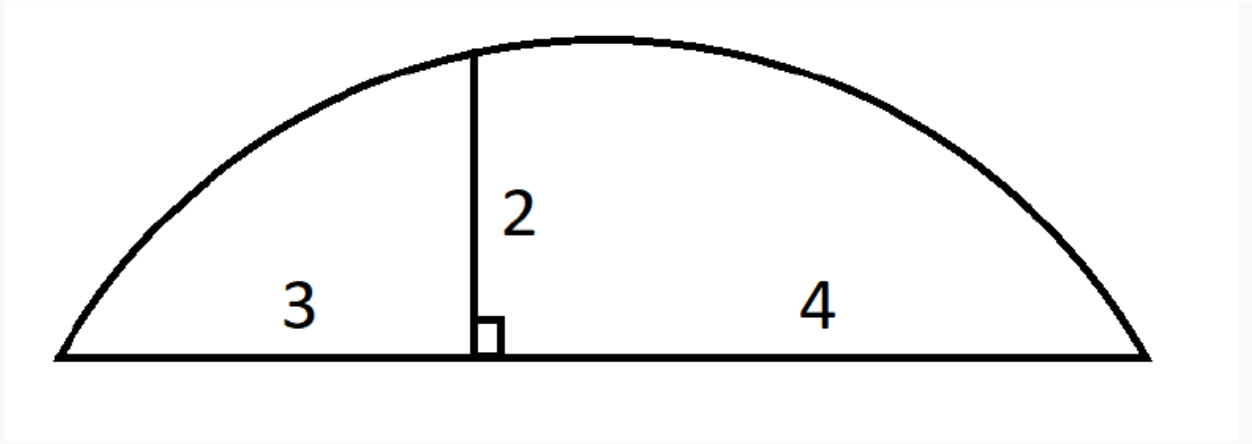
Question

The following diagram shows a chord drawn across a circle. From a point on the chord, a perpendicular line is drawn to another point on the same circle. Distances are shown in the diagram above.

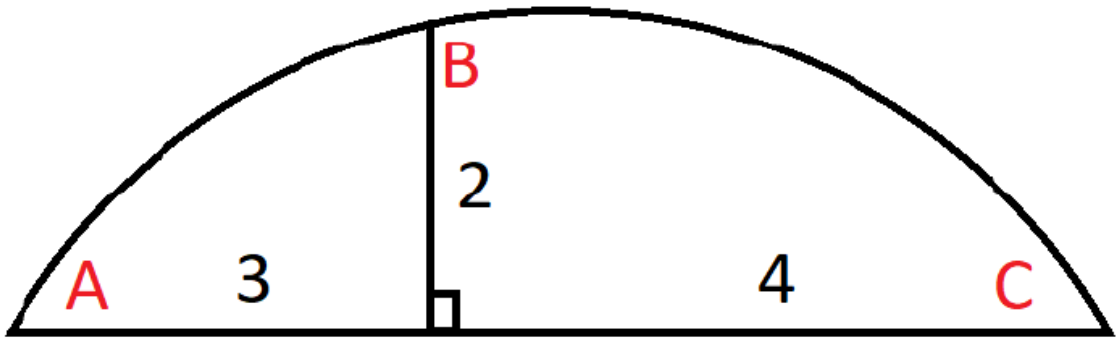


What is the radius of the circle?

Answer

There are a number of solutions possible to this puzzle. The following may not be the shortest, but requires no formulas other than the Pythagorean.

First, let's assign letters to the three points where the lines touch the circle:



Next, let's assign coordinates to those points, as follows:

- A = (-3,0)
- B = (0,2)
- C = (4,0)

Using simple geometry, here are the equations for two lines:

- AB: $y = \frac{3}{2}x + 2$
- BC = $-\frac{1}{2}x + 2$

Next, fine the midpoints between both sets of points:

- A and B: $(-\frac{3}{2}, 1)$
- B and C: (2,1)

Next, find equations to the lines that are perpendicular to AB and BC that run through those midpoints:

- Perpendicular line to AB running through $(-\frac{3}{2}, 1)$: $y = -\frac{3}{2}x - \frac{5}{4}$
- Perpendicular line to BC running through (2,1) : $y = 2x - 3$

To find the point where these two perpendiculars meet, set them equal to each other:

$$-\frac{3}{2}x - \frac{5}{4} = 2x - 3$$

We easily find that they meet at $(\frac{1}{2}, -2)$

That is the center of the circle.

We can now use the pythagorian formula with that point and either A, B, or C to find the radius.

Choosing B, arbitrarily, we find that

$$r = \sqrt{4^2 + (\frac{1}{2})^2} = \sqrt{65} / 2 =\sim 4.03112887414927$$