

Q: You are given:

- Old McDonald has a farm.
- On that farm are sheep.
- The sheep are let into a pasture where the grass is 2" high.
- Grass grows at a constant rate.
- The sheep eat all the time.
- The time it takes to eat a blade of grass is proportional to its height.
- A sheep will eat a blade of grass completely and then move onto the next blade.
- Once a blade of grass is eaten completely, it does not grow back.
- It takes the sheep three days to clear the first acre of grass.

How long does it take the sheep to clear the second acre of grass?

See next page for the answer.

Answer: 7.5 days

See next page for the solution.

Solution:

The number of blades of grass the sheep can eat in the first three days can be expressed as:

$$\int_0^3 \frac{1}{t+2} dt =$$

$$\ln(t+2) \text{ for } t \text{ from } 0 \text{ to } 3 =$$

$$\ln(5) - \ln(2) = \ln\left(\frac{5}{2}\right)$$

Let x be the number of days required after the first three days to reach the second acre of grass. The grass consumed in that time can be expressed as:

$$\int_3^{3+x} \frac{1}{t+2} dt =$$

$$\ln(t+2) \text{ for } t \text{ from } 3 \text{ to } x+3 =$$

$$\ln(x+3) - \ln(3) = \ln\left(\frac{x+3}{3}\right)$$

The number of blades of grass in an acre is constant. So, the number of blades eaten in the first three days equals the number eaten in the next x days after that. Thus, we can equate the two integrals:

$$\ln\left(\frac{5}{2}\right) = \ln\left(\frac{x+3}{3}\right)$$

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$$15 = 2x + 6$$

$$2x = 9$$

$$x = \frac{9}{2} = 4.5 \text{ days}$$