How to Analyze the Even, Odd, Downtown/Uptown, Horn, and Citywide bets in Repeater Bets Plus

The purpose of this document is to show how to analyze certain bets in the game Repeater Bets Plus. This game is played with two six-sided dice. The following are the bets this document covers.

Even Bet – To win this bet at least one of the following events must occur before the shooter rolls a total of seven. Wins pay 12 for 1.

- Two totals of 2
- Four totals of 4
- Six totals of 6
- Six totals of 8
- Four totals of 10
- Two totals of 12

Odd Bet – To win this bet at least one of the following events must occur before the shooter rolls a total of seven. Wins pay 18 for 1.

- Three totals of 3
- Five totals of 5
- Five totals of 9
- Three totals of 11

Downtown -- To win this bet at least one of the following events must occur before the shooter rolls a total of seven. Wins pay 12 for 1.

- Two totals of 2
- Three totals of 3
- Four totals of 4
- Five totals of 5
- Six totals of 6

Uptown -- To win this bet at least one of the following events must occur before the shooter rolls a total of seven. Wins pay 12 for 1.

- Two totals of 12
- Three totals of 11
- Four totals of 10
- Five totals of 9
- Six totals of 8

Horn -- To win this bet at least one of the following events must occur before the shooter rolls a total of seven. Wins pay 12 for 1.

- Two totals of 2
- Three totals of 3
- Three totals of 11
- Two totals of 12

Citywide -- To win this bet at least one of the following events must occur before the shooter rolls a total of seven. Wins pay 12 for 1.

- Two totals of 2
- Three totals of 3
- Four totals of 4
- Five totals of 5
- Six totals of 6
- Six totals of 8
- Five totals of 9
- Four totals of 10
- Three totals of 11
- Two totals of 12

The probability of any of these bets winning is the same if the time between roles were distributed according to the exponential distribution with a mean of 1.

Recall from the Poisson distribution the probability of exactly n events occurring in a given period of time, with a mean of m, is:

$$\frac{e^{-m} \times m^{n}}{n!}$$

Let me review the probability of any given dice total with two dice:

- 2 or 12: 1/36
- 3 or 11: 1/18
- 4 or 10: 1/12
- 5 or 9: 1/9
- 6 or 8: 5/36
- 7: 1/6

The probability of a seven occurring, for the first time, after x units of time, is (1/6)*exp(-t/6).

Even Bet

A seven has to be rolled eventually. When it does, the bet will lose if and only if all of the following conditions are true:

0 or 1 totals of 2

0 to 3 totals of 4

0 to 5 totals of 6

0 to 5 totals of 8

0 to 3 totals of 10

0 or 1 totals of 12

Let f(x) = Probability the Even bet loses after x units of time. Let pr(y) = probability of event y.

f(x) = (1/6) * pr(no sevens in x units of time) * pr(1 or less totals of 12) * pr(3 or less totals of 4) * pr(5 or less totals of 6) * pr(5 or less totals of 8) * pr(3 or less totals of 10) * pr(1 or less totals of 12).

The probabilities for the events on a total of 8, 10, and 12 are the same as those for 2, 4, and 6, so we can rewrite as:

f(x) = (1/6) * pr(no sevens in x units of time) * (pr(1 or less totals of 12) * pr(3 or less totals of 4) * pr(5 or less totals of 6))^2

Using the Poisson distribution, we can rewrite as follows:

f(x) = (1/6)*exp(-x/6)*(exp(-x/36)*(1+(x/36))*exp(-x/12)*(1+(x/12)+(x/12)^2/2+(x/12)^3/6)*exp(-5x/36)*(1+(5x/36)+(5x/36)^2/2+(5x/36)^3/6+(5x/36)^4/24+(5x/36)^5/120))^2

Again, this is the probability of losing. To solve, integrate this function from 0 to infinity, because the bet could lose after any amount of time. Using an integral calculator (I recommend the one at integral-calculator.com), results in a probability of losing of

5177968147808912897/5540271966595842048 = apx. 0.9346054090897739.

The probability of winning is thus 1 - 0.9346054090897739 = 0.0653945909102263.

Odd Bet

Let me go faster through the rest of the bets. The probability the Odd bet loses after exactly x units of time is

(1/6)*exp(-x/6)*(exp(-x/18)*(1+(x/18)+(x/18)^2/2)*exp(-x/9)*(1+(x/9)+(x/9)^2/2+(x/9)^3/6+(x/9)^4/24))^2

Integrating that from 0 to infinity yields a probability of losing of

30026163533/31381059609 = apx. 0.9568244000399713

This makes the probability of winning 0.0431755999600287

Downtown/Uptown Bets

The probability the Downtown bet loses after exactly x units of time is

(1/6)*exp(-x/6)*

(exp(-x/36)*(1+(x/36)))*

(exp(-x/18)*(1+(x/18)+(x/18)^2/2))*

(exp(-x/12)*(1+(x/12)+(x/12)^2/2+(x/12)^3/6))*

(exp(-x/9)*(1+(x/9)+(x/9)^2/2+(x/9)^3/6+(x/9)^4/24))*

(exp(-5x/36)*(1+(5x/36)+(5x/36)^2/2+(5x/36)^3/6+(5x/36)^4/24+(5x/36)^5/120))

Integrating that from 0 to infinity yields a probability of losing of

12620691321898496/13349464742886867= apx. 0.9454080418185537

This makes the probability of winning 0.0545919581814487

The Uptown bet is the mirror image of the Downtown bet, thus the answer and solution are the same.

Horn Bet

The probability the Horn bet loses after exactly x units of time is

(1/6)*exp(-x/6)*(exp(-x/36)*(1+(x/36))*exp(-x/18)*(1+(x/18)+(x/18)^2/2))^2

Integrating that from 0 to infinity yields a probability of losing of

3239/3456 = apx. 0.9372106481481481

This makes the probability of winning 0.0627893518518519.

Citywide Bet

The probability the Citywide bet loses after exactly x units of time is

(1/6)*exp(-x/6)*
((exp(-x/36)*(1+(x/36)))*
(exp(-x/18)*(1+(x/18)+(x/18)^2/2))*
(exp(-x/12)*(1+(x/12)+(x/12)^2/2+(x/12)^3/6))*
(exp(-x/9)*(1+(x/9)+(x/9)^2/2+(x/9)^3/6+(x/9)^4/24))*
(exp(-5x/36)*(1+(5x/36)+(5x/36)^2/2+(5x/36)^3/6+(5x/36)^4/24+(5x/36)^5/120)))^2
Integrating that from 0 to infinity yields a probability of losing of

1037684062429515325317113056769 87/1141980881111376812748576412139 52 = apx. 0.9086702584894769

This makes the probability of winning 0.0913297415105240.