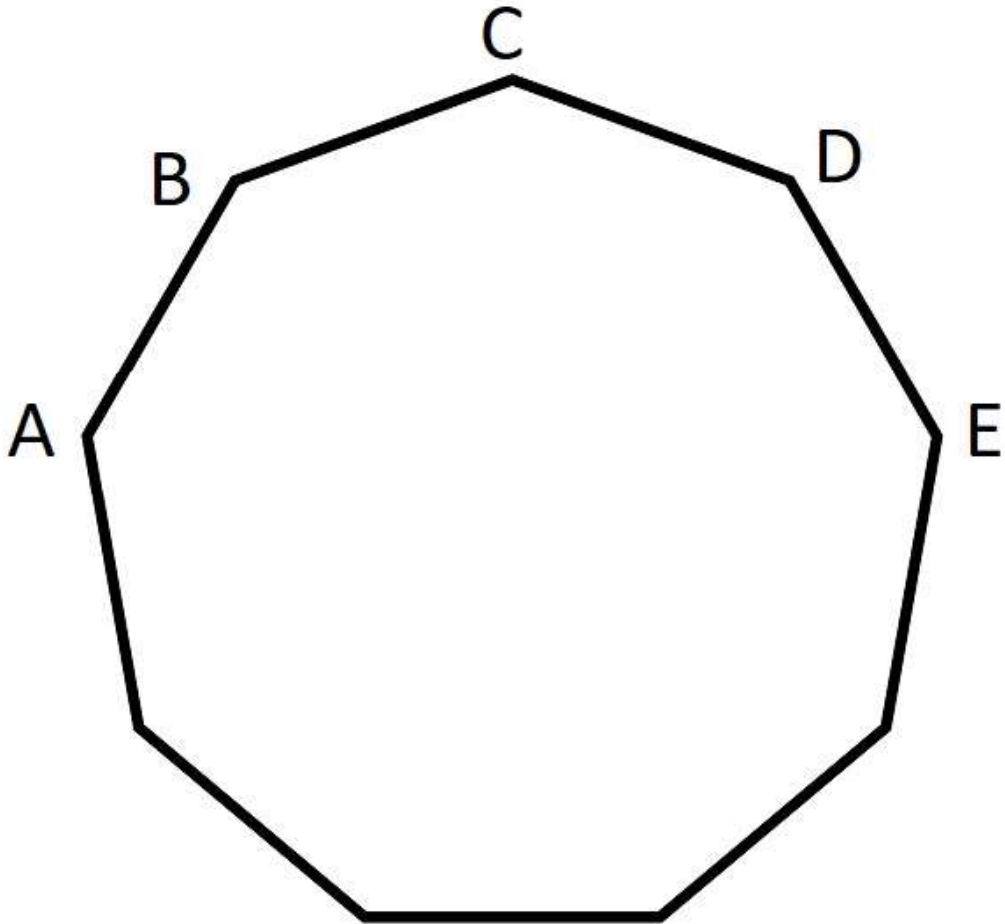


Question:



The figure above is a nonagon with side length one.

Which is more, $AB+AC$ or AE ?

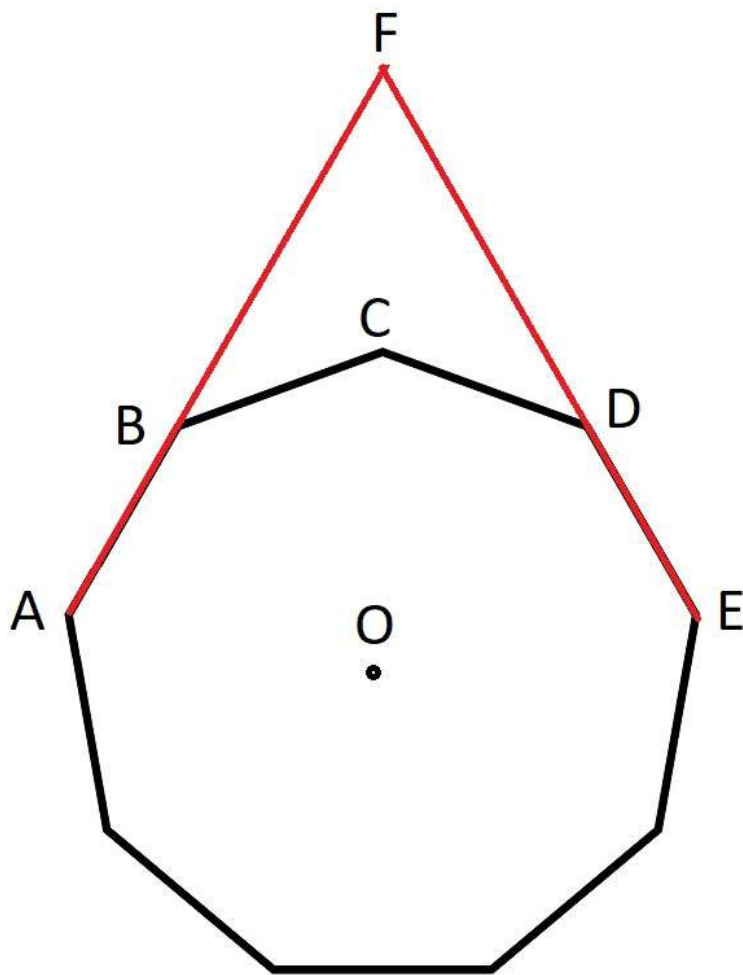
Answer: They are the same

Solution:

Add the following points to the diagram:

O = Center of nonagon

F = Intersection of the lines created by extending AB and DE.



Consider the triangle ABO. If you divide the nonagon into nine triangular slices, each slice will touch the center with an angle of $360/9 = 40$. The other two angles of each slice will be $(180-40)/2 = 70$. Thus, $\angle ABO = 70$.

$\angle ABC = 2 * \angle ABO = 140$, since triangles ABO and BCO are the same.

$\angle BAC$ and $\angle BCA$ are both equal to $(180-140)/2 = 20$.

$\angle BCD = \angle ABC = 140$.

Next, consider $\angle ACE$.

$\angle ACE = \angle BCD - 2 * \angle CBD = 140 - 2 * 20 = 100$.

$\angle CAE = \angle CEA = (180-100)/2 = 40$.

Next, consider $\angle FAE$.

$\angle FAE = \angle BAC + \angle CAE = 20 + 40 = 60$

So, triangle AEF is equilateral! This is the “ah-ha” moment we need.

Since AEF is equilateral, $AE = AF = AB + BF$.

What is BF? Since $\angle AFE = 60$, so must $\angle BFD$, so triangle BDF is also equilateral. Thus, the three sides of BDF are the same: $BD = BF = DF$.

$AE = AB + BF = AB + BD = AB + AC$.

Q.E.D.