Question

For what value is $x^{\frac{1}{x}}$ a maximum?

Answer

The answer is e =~ 2.71828182845905

Solution

Let y=$x^{\frac{1}{x}}$

ln(y) = ln($x^{\frac{1}{x}}$)

ln(y) = $\frac{1}{x}$ ln(x)

To find the maximum or minimum value of any function we take the derivative.

dy/dx (ln(y)) = ($\frac{1}{x}$ ln(x)) d/dx

On the left side, remember the derivative of a function is the derivative of the expression times the derivative of what's inside.

On the right side, use the product rule.

$\frac{1}{y}$ * dy/dx = $\frac{1}{x}$ * $\frac{1}{x}$ - $\frac{1}{x^2}$ * ln(x)

dy/dx = y * ($\frac{1}{x^2}$ - $\frac{\ln(x)}{x^2}$)

= $x^{\frac{1}{x}}$ * ($\frac{1}{x^2}$ - $\frac{\ln(x)}{x^2}$)

= $x^{\frac{1}{x}}$ * $\frac{1}{x^2}$ * (1-ln(x))

= $x^{\left(\frac{1}{x}-2\right)} * (1 - \ln(x))$

To find the root set this equal to zero. This will be true when…

$1 - \ln(x) = 0$

Ln(x) = 1

x = e

Acknowledgements:

blackpenredpen YouTube channel:
https://www.youtube.com/watch?v=QQWDpBfWhp8