Q: You have a 5' ladder. You wish to lean it against a wall as high as possible. However, there is a 1x1x1 cubic foot box you must put the ladder over. No, you can't move the box. What is the maximum height the top of the ladder can reach?

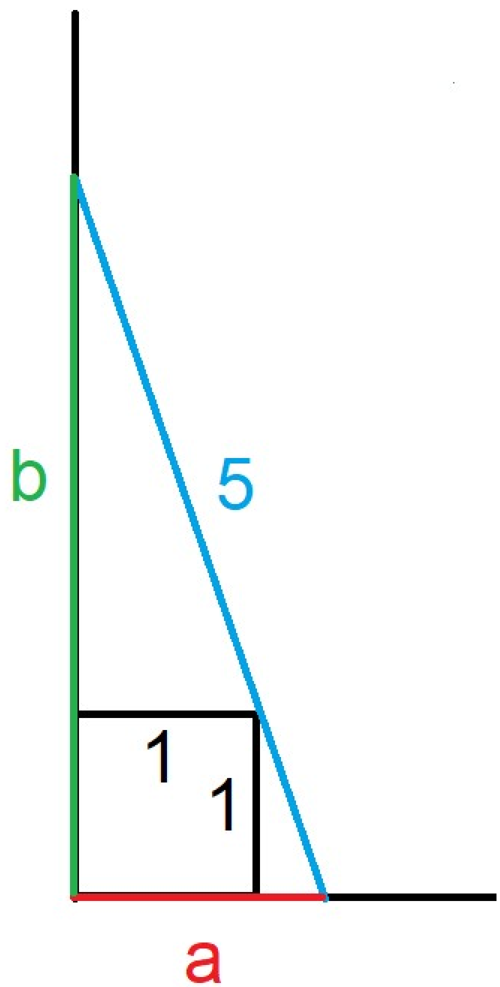
A: $(1+\sqrt{26} + \sqrt{23 - 2\sqrt{26}}) / 2 =\sim 4.838501$

Solution:

First, let's let:

a = distance from bottom of ladder to the wall

b = height of top of ladder from the ground



From similar triangles we know:

b/a = (b-1)/1

Cross multiply:

b = ab – a

a+b = ab

From the Pythagorean formula we know:

a^2 + b^2 = 25

$(a + b)^2$ – 2ab = 25

Let c = 25

c^2 – 2c – 25 = 0

Using the quadratic formula, we get:

c = a b = 1 + $\sqrt{26}$ =~ 6.099020

Recall, a + b = ab

So, a + b = 1 + $\sqrt{26}$

b = 1 + $\sqrt{26} - a$

We also have:

a b = 1 + $\sqrt{26}$

a (1 + $\sqrt{26} - a$) $=$ 1 + $\sqrt{26}$

Let d = 1 + $\sqrt{26}$

a + (d - a) = d

a d - a^2 = d

a^2 – a d + d = 0

Using the quadratic formula, we get:

a = (d +/- $\sqrt{d^2 - 4d}$) / 2

Substitute d = 1 + $\sqrt{26}$

$$a = \frac{1 + \sqrt{26} \pm \sqrt{(1 + \sqrt{26})^2 - 4 \times 4(1 + \sqrt{26})}}{2}$$

After some simplification:

$$a = \frac{1+ \sqrt{26} \pm \sqrt{23-2\sqrt{26}}}{2}$$

Of the + and - , we should choose the -, because a < b, because we want the ladder to stand up as high as possible. So,

$$a = \frac{1+ \sqrt{26} - \sqrt{23-2\sqrt{26}}}{2} = 1.260518$$

Knowing a and that a+b = 1 + $\sqrt{26}$, simple algebra gives us:

$$b = \frac{1+ \sqrt{26} - \sqrt{23-2\sqrt{26}}}{2} =\sim 4.838501$$