Q: What is i^i?

A:

(1) $$i^i = e^{\ln(i^i)} = e^{i \ln(i)}$$

Next, recall Euler's formula:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

If $\theta = \frac{\pi}{2}$ then,

$$e^{i\frac{\pi}{2}} = \cos(\tfrac{\pi}{2}) + i\sin(\tfrac{\pi}{2})$$

$$= 0 + i = i$$

So, we have $i = e^{i\frac{\pi}{2}}$

Let's substitute that value of i for the i in the ln(i) in equation (1):

$$i^i = e^{i\ln(i)} = e^{i\ln(e^{i\frac{\pi}{2}})}$$

$$= e^{i\, i\frac{\pi}{2}}$$

$$= e^{-\frac{\pi}{2}}$$

=
0.207879576350761908546955619834978770033877841631769608075135883055419877285482139788600277865426035340521773307235021808190619730374663986999112631786412057317177795200674337664954224638192973743053870376005189066303304970051900555620047586620529435183442…

My thanks to Matt Parker for this solution to this problem at
https://www.youtube.com/watch?v=9tlHQOKMHGA

For the answer to 100 digits, I used WizCalc:

https://wizardofodds.com/games/math/calculator/