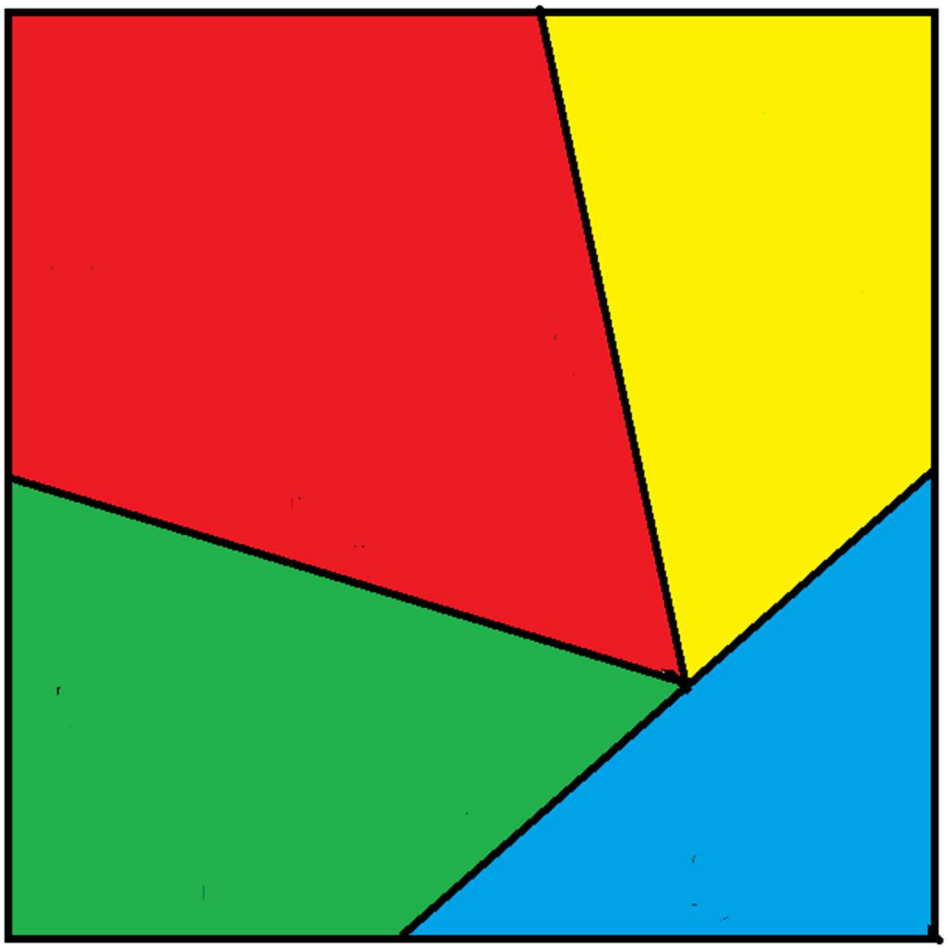
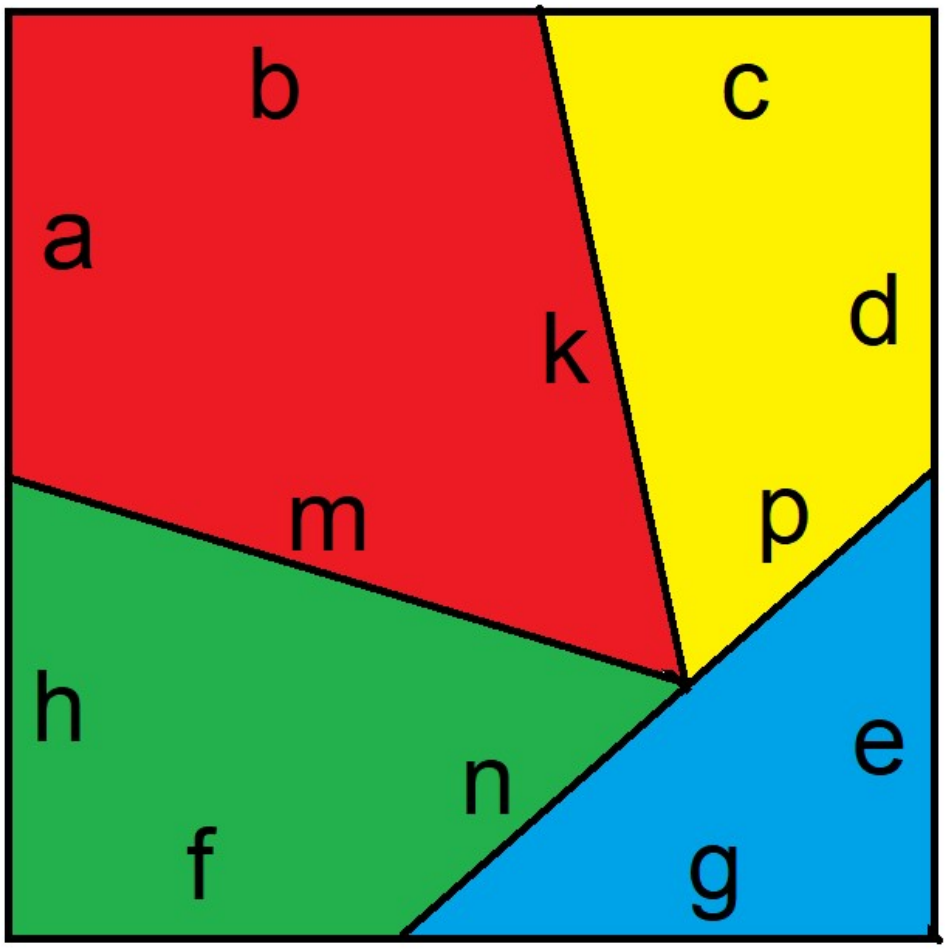
Question: Rearrange the following pieces to make an equilateral triangle. Flipping pieces is not needed. Assuming the sides of the square are length 1.

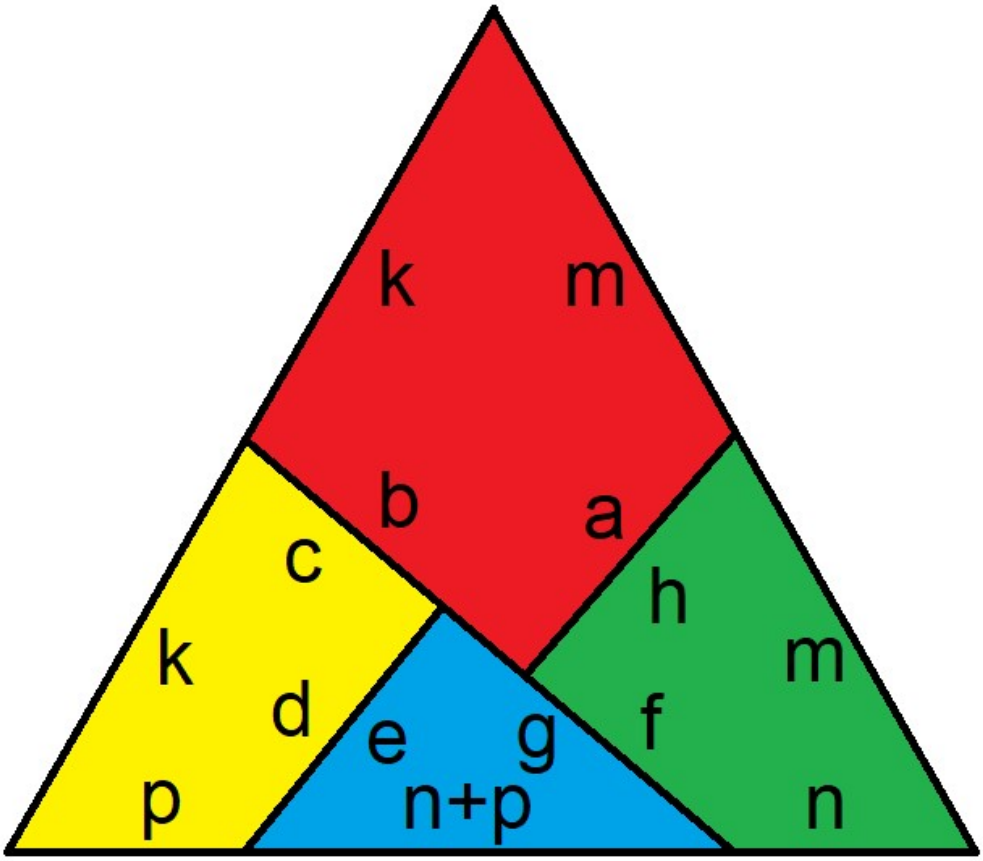


Solution

First, let's name the sides of the pieces as follows:



The solved puzzle looks like this:



We can see the following equivalences:

a=h

d=e

We know from the square that:

a+h = 1

d+e=1

It's easy to see then that a=h=d=e=1/2.

Next, let's solve for m. We're given it's an equilateral triangle, so every side of the triangle must have length 2m. Using the Pythagorean formula, we can find the height of the triangle is $\sqrt{3}$ m. The area of the tringle is base*height/2 =1

2m * $\sqrt{3}$ m / 2 = 1

$\sqrt{3}\, m^2$= 1

m = 1/$\sqrt{\sqrt{3}}$ =~ 0.759836

Another side of the triangle is 2k, so k also equals 1/$\sqrt{\sqrt{3}}$.

The bottom side of the triangle is:

p + n + p + n = 2/$\sqrt{\sqrt{3}}$

2(n+p) = 2/$\sqrt{\sqrt{3}}$

n + p = 1/$\sqrt{\sqrt{3}}$

Next, consider the right triangle with sides e, g, n+p.

We know e=1/2 and n+p = 1/$\sqrt{\sqrt{3}}$

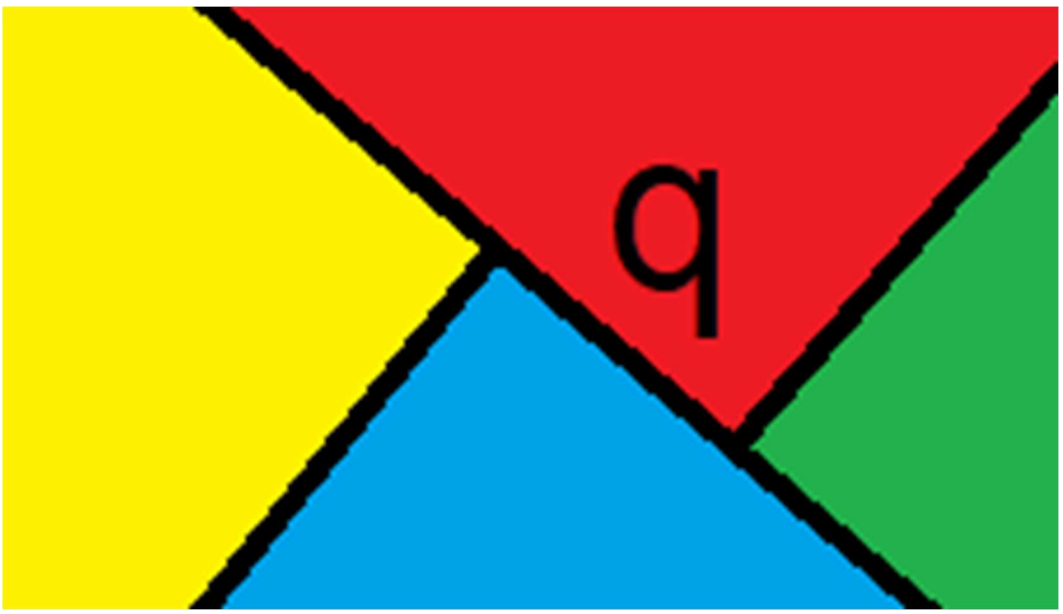
A few steps with the Pythagorean formula gives us:

g = $\sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}}$ =~ 0.572145

f+g = 1, so:

$f = 1 - g = 1 - \sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}} =\sim 0.427855$

Next, let's introduce another variable q, for this short piece:



From the image of the whole triangle, we see:

$f + q = g$

$q = g - f$

$$q = \sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}} - \left(1 - \sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}}\right) = 2\sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}} - 1 =\sim 0.144291$$

We now know from the square $f+g = 1$

We just solved for $g-f = q$. So, $g = f+q$.

Substituting $g=f+q$ into $f+g = 1$ gives us:

$f + (f+q) = 1$

$2f = 1-q$

$$f = (1-q)/2 = (1 - (2\sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}} - 1)) / 2$$

$$= (2 - 2\sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}}) / 2 = 1 - \sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}} =\sim 0.427855$$

From the big triangle, we know c+q = b

From the square, we know b+c = 1

Simple algebra gives us b=(1+q)/2.

We have solved for q already, so:

$$b = \sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}} =\sim 0.572145$$

Again, b+c = 1, so we can solve for c as:

$$c = 1-b = 1 - \sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}} =\sim 0.427855$$

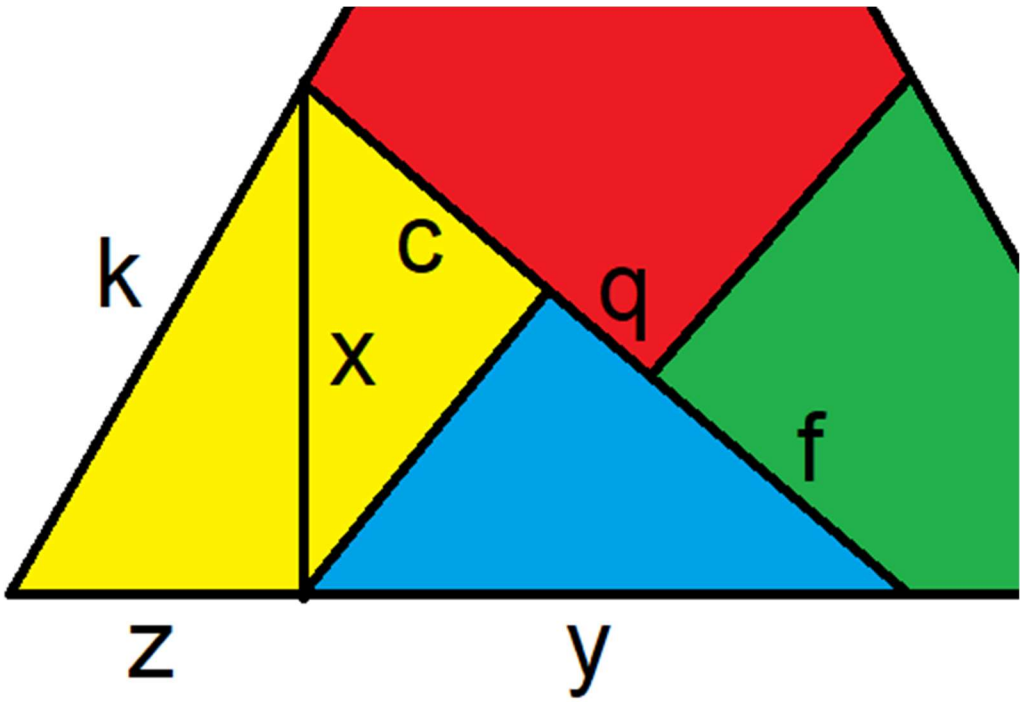
We have seen this number before, f.

So:

f = c =~ 0.427855, which also easily leads to

b = g =~ 0.572145

Next, let's work with the following two pieces. Note the new variables I have introduced, where x is the height of the triangle.



Please note that we can't assume p = z and n+p = y, although it certainly looks like it.

We do know z+y = p+n+p = n+2p.

We also know triangle xkz is a 30-60-90 triangle. When you know one piece of a 30-60-90 triangle, it's easy to find the others. In this case, we know k = $1/\sqrt{\sqrt{3}}$.

Using the Pythagorean formula, we can get:

z = $1/(2\sqrt{\sqrt{3}})$ =~ 0.379918

x = $\sqrt{\sqrt{3}}/2$ =~ 0.658037

Next, let's solve for the side of the triangle c+q+f =

c + g

We have shown already that g=b so:

c +g = c + b.

We know from the square that b+c = 1, so c+q+f = 1

Next, now that we know x, we can solve for y, again using the Pythagorean formula.

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

Some simple algebra gives us:

y = $(\sqrt{4 - \sqrt{3}})/2$ =~ 0.752986

We can also express z+y as n+2p.

We know z and y, so:

z + y = $1/(2\sqrt{\sqrt{3}}) + (\sqrt{4 - \sqrt{3}})/2$

We know from way back:

n + p = $1/\sqrt{\sqrt{3}}$

So,

n + 2p = (n+p) + p = $1/\sqrt{\sqrt{3}}$ + p

Equating that to z+y and some simple algebra gives us:

$$p = \frac{\sqrt{4\sqrt{3}-3}-1}{2\sqrt{\sqrt{3}}} =\sim 0.373068$$

We already know n+p =$1/\sqrt{\sqrt{3}}$ and we now know p, so simple algebra gives us:

$$n = \frac{3-\sqrt{4\sqrt{3}-3}}{2\sqrt{\sqrt{3}}} =\sim 0.386768$$

So, that is the final letter solved. As a summary:

a	0.500000000000000
b	0.572145321740575
c	0.427854678259425
d	0.500000000000000
e	0.500000000000000
f	0.427854678259425
g	0.572145321740575
h	0.500000000000000
k	0.759835685651592
m	0.759835685651592
n	0.386767938902275
p	0.373067746749317
q	0.144290643481149
x	0.658037006476246
y	0.752985589575113
z	0.379917842825796