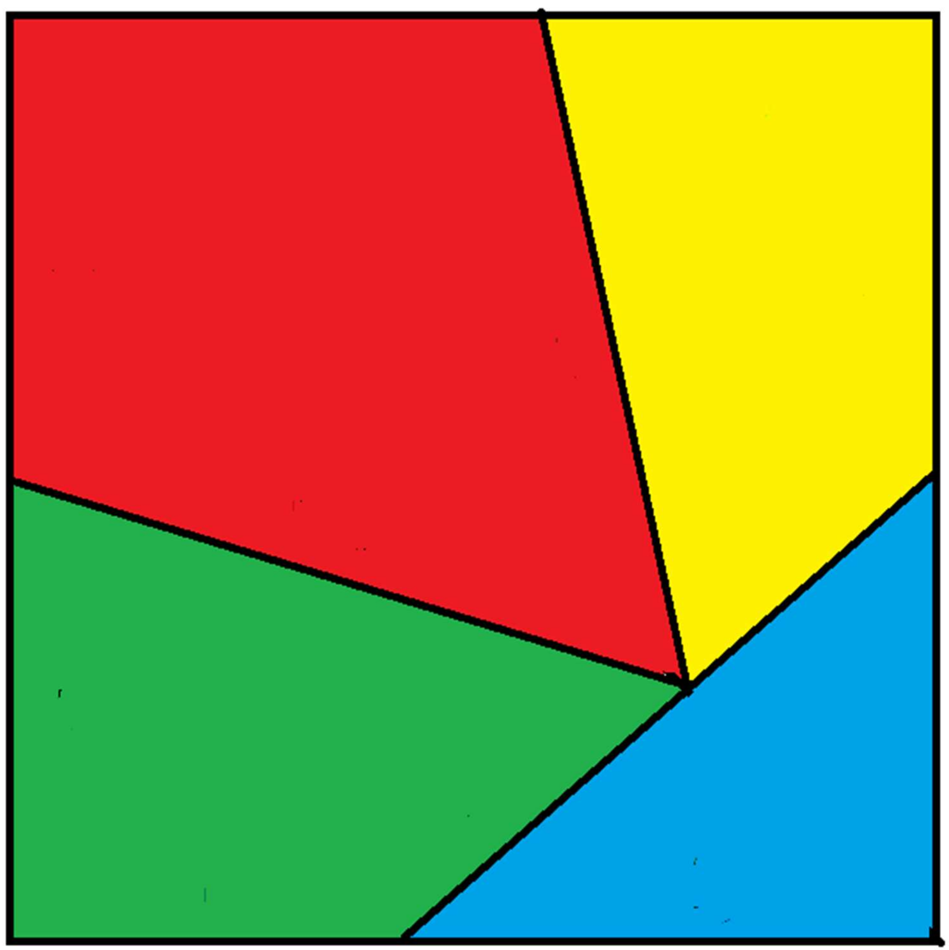
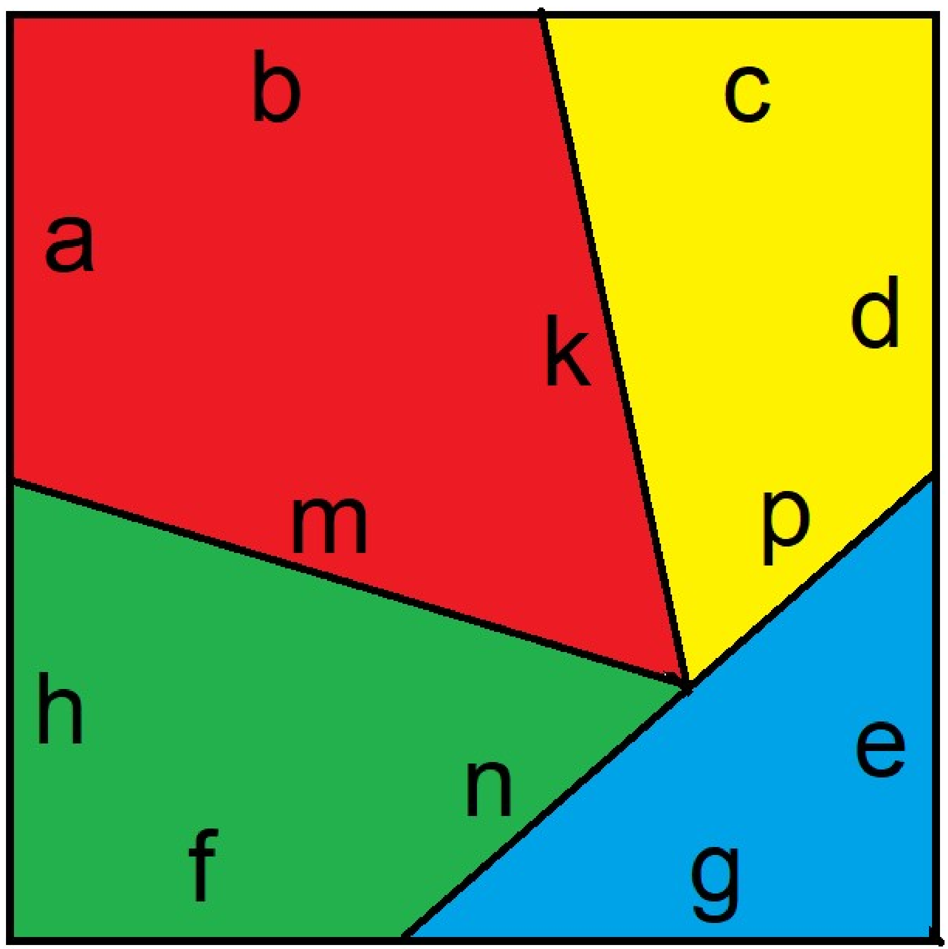
Question: Rearrange the following pieces to make an equilateral triangle. Flipping pieces is not needed. Assuming the sides of the square are length 1.

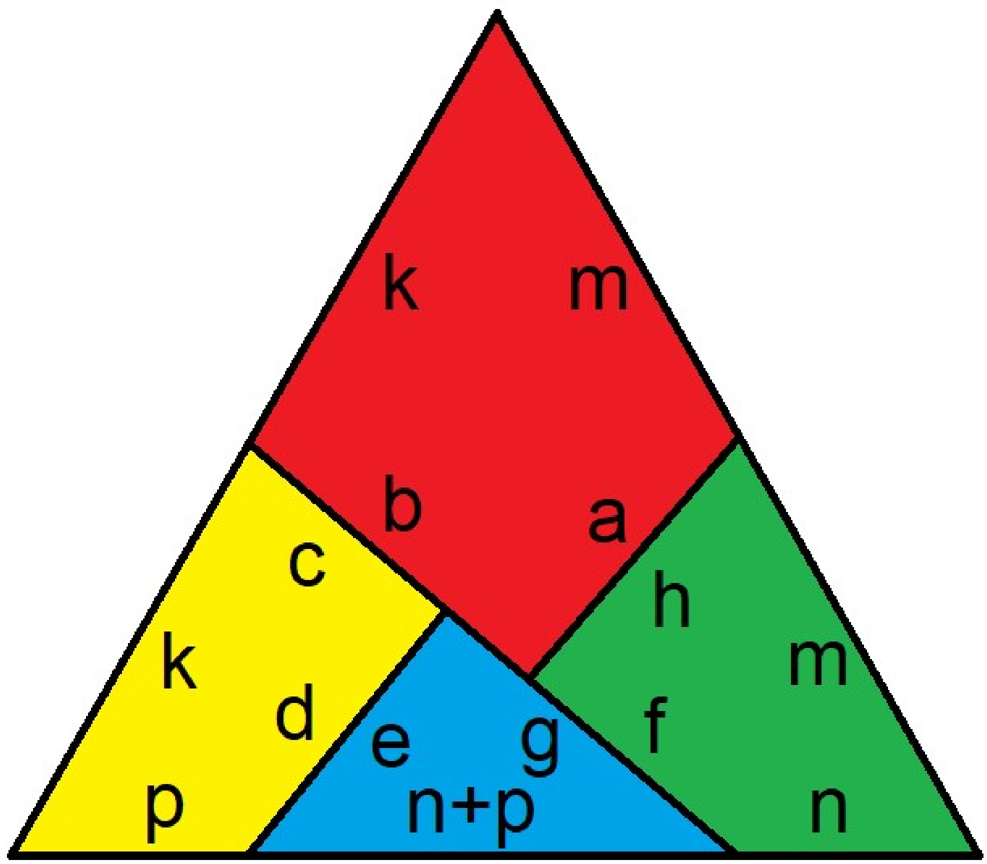


Solution

First, let’s name the sides of the pieces as follows:



The solved puzzle looks like this:



We can see the following equivalences:

a=h

d=e

We know from the square that:

a+h = 1

d+e=1

It's easy to see then that a=h=d=e=1/2.

Next, let's solve for m. We're given it's an equilateral triangle, so every side of the triangle must have length 2m. Using the Pythagorean formula, we can find the height of the triangle is $\sqrt{3}$ m. The area of the tringle is base*height/2 =1

2m * $\sqrt{3}$ m / 2 = 1

$\sqrt{3}\, m^2$= 1

m = 1/$\sqrt{\sqrt{3}}$ =~ 0.759836

It is also clear that 2k is the same side of the same equilateral triangle, so:

k = m = 1/$\sqrt{\sqrt{3}}$ =~ 0.759836

Another side of the triangle is 2k, so k also equals 1/$\sqrt{\sqrt{3}}$.

The bottom side of the triangle is:

p + n + p + n = 2/$\sqrt{\sqrt{3}}$

2(n+p) = 2/$\sqrt{\sqrt{3}}$

n + p = 1/$\sqrt{\sqrt{3}}$

Next, consider the right triangle with sides e, g, n+p.

We know e=1/2 and n+p = 1/$\sqrt{\sqrt{3}}$

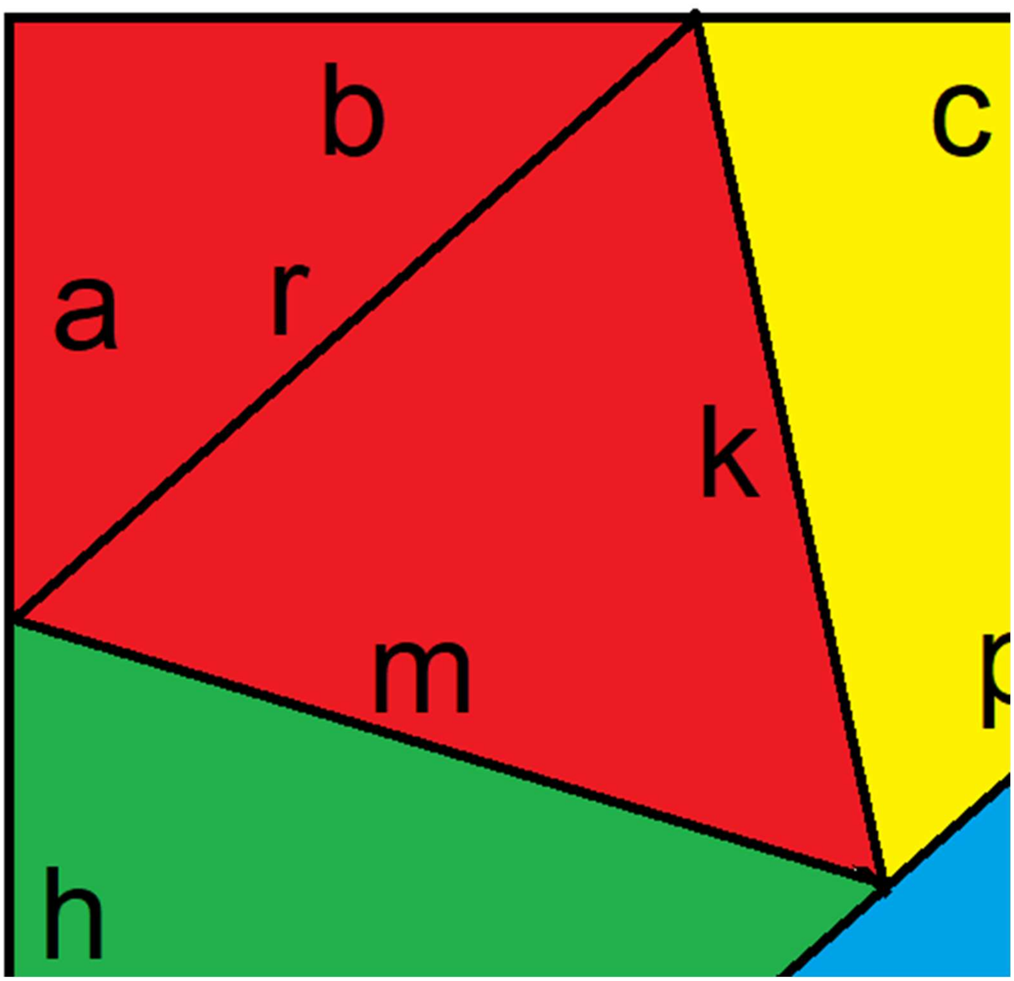
A few steps with the Pythagorean formula gives us:

g = $\sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}}$ =~ 0.572145

f+g = 1, so:

f = 1 – g = 1 - $\sqrt{\frac{4-\sqrt{3}}{4\sqrt{3}}}$ =~ 0.427855

Next, let me introduce a new term, r:



We already showed that k = m = $1/\sqrt{\sqrt{3}}$

By the conditions of the puzzle, the angle formed between sides k and m is 60 degrees. Since k=m and the angle between them is 60 degrees, the whole triangle must be an equilateral triangle. That makes:

$r = k = m = 1/\sqrt{\sqrt{3}}$

Now that we know a and r, we can solve for b:

$a^2 + b^2 = r^2$

$b^2 = r^2 - a^2$

$= 1/\sqrt{3} - ¼$

$= \frac{4-\sqrt{3}}{4\sqrt{3}}$

$b = \frac{\sqrt{4-\sqrt{3}}}{2\sqrt{\sqrt{3}}}$ =~ 0.572145

Note, this is the same as g.

Since b+c = 1, we easily find

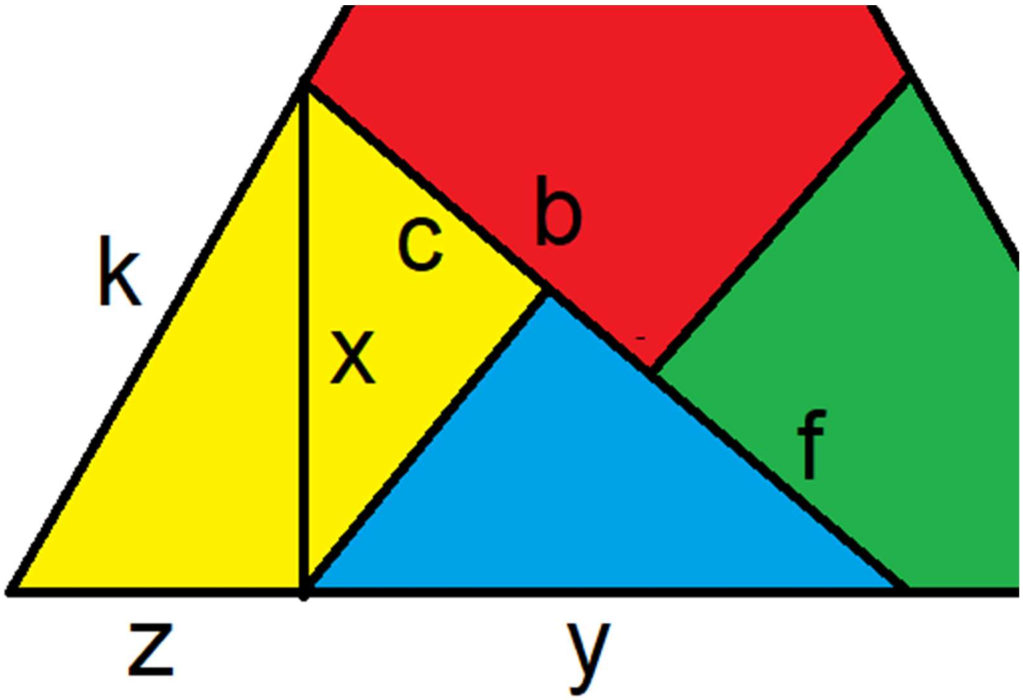
$c = 1 - b = 1 - \frac{\sqrt{4-\sqrt{3}}}{2\sqrt{\sqrt{3}}} = \sim 0.427855 = f$

As a recap:

$c = f = 1 - \frac{\sqrt{4-\sqrt{3}}}{2\sqrt{\sqrt{3}}}$

$b = g = \frac{\sqrt{4-\sqrt{3}}}{2\sqrt{\sqrt{3}}}$

Next, let's work with the yellow and blue two pieces. Note the new variables I have introduced, where x is the height of the triangle.



Please note that we can't assume p = z and n+p = y, although it certainly looks like it.

We do know z+y = p+n+p = n+2p.

We also know triangle xkz is a 30-60-90 triangle. When you know one piece of a 30-60-90 triangle, it's easy to find the others. In this case, we know k = $1/\sqrt{\sqrt{3}}$.

Using the Pythagorean formula, we can get:

z = $1/(2\sqrt{\sqrt{3}})$ =~ 0.379918

x = $\sqrt{\sqrt{3}}/2$ =~ 0.658037

Next, let's solve for y. We know:

$x^2 + y^2 = (b + f)^2$

$y^2 = (b + f)^2 - x^2$

As a reminder, b + c = 1 and c=f.

So, b + f = 1

$y^2 = 1 - x^2$

$y^2 = 1 - \sqrt{3}/4$

y = $\sqrt{1 - \sqrt{3}/4} = \frac{\sqrt{4 - \sqrt{3}}}{2}$

We know z and y, so:

z + y = $1/(2\sqrt{\sqrt{3}}) + (\sqrt{4 - \sqrt{3}})/2$

We know from way back:

n + p = $1/\sqrt{\sqrt{3}}$

So,

n + 2p = (n+p) + p = $1/\sqrt{\sqrt{3}}$ + p

Equating that to z+y and some simple algebra gives us:

p = $\frac{\sqrt{4\sqrt{3}-3}-1}{2\sqrt{\sqrt{3}}}$ =~ 0.373068

We already know n+p =$1/\sqrt{\sqrt{3}}$ and we now know p, so simple algebra gives us:

n = $\frac{3-\sqrt{4\sqrt{3}-3}}{2\sqrt{\sqrt{3}}}$ =~0.386768

So, that is the final letter solved!

As a summary:

a	0.500000000000000
b	0.572145321740575
c	0.427854678259425
d	0.500000000000000
e	0.500000000000000
f	0.427854678259425
g	0.572145321740575
h	0.500000000000000
k	0.759835685651592
m	0.759835685651592
n	0.386767938902275
p	0.373067746749317
r	0.759835685651592
x	0.658037006476246
y	0.752985589575113
z	0.379917842825796

My thanks to “chevy” for his help with this solution.

This problem is discussed in my forum at:

https://wizardofvegas.com/forum/questions-and-answers/math/37648-haberdashers-puzzle/