

Question: A microbe, let's call it Covid-20, spawns a new microbe at an average rate of one per day in the lungs. New microbes all spawn at the same rate. The expected time between new microbes forming is inversely proportional to the number of microbes.

Once a microbe enters your lungs, how many microbes will you have after seven days?

Answer: $e^7 = \text{apx. } 1096.63315842845859926372$.

Solution:

Let's call $m(t)$ = Covid-20 microbes at time t .

We're given that $m(0)=1$.

For any number of microbes m , the rate at which new microbes will come along is m . We can express this mathematically as:

$$\frac{dm}{dt} = m.$$

Multiplying both sides by dt , we get:

$$dm = m dt$$

Let's move the m term to the left side by dividing both sides by m .

$$(1/m) dm = dt$$

Next, let's integrate both sides:

$$\ln(m) = t + c, \text{ where } c \text{ is the constant of integration.}$$

We're given that at time 0 there is one microbe. Putting that initial condition into the equation above:

$$\ln(1) = 0 + c$$

$$0 = 0 + c$$

$$c = 0$$

So now we have:

$$\ln(m) = t$$

$$m = e^7$$

So, the number of microbes at time $t=7$, is $e^7 = \text{apx. } 1096.6331584284585992637$