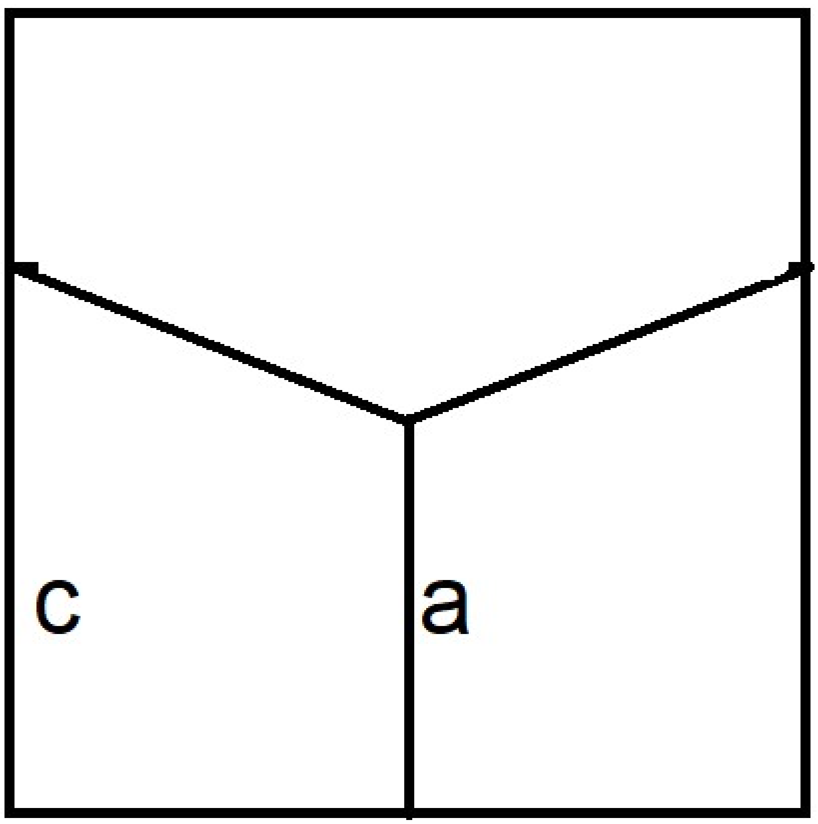
Question: A farmer has a 1x1 square piece of land. He wishes to divide it equally among his three children. He may use no more than three straight sections of fencing to divide the property. How should he do it to minimize the length of the fencing?

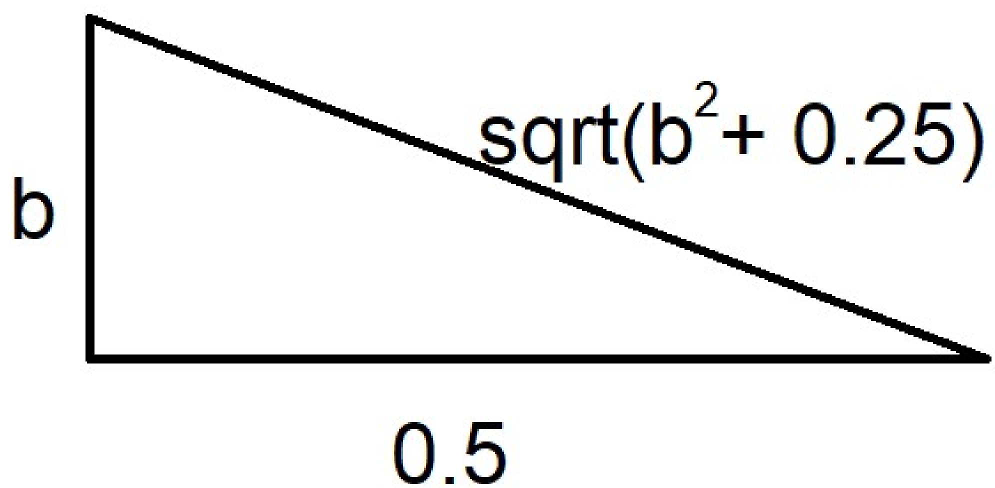
Hint: Divide the square in the following manner.



Solution:

Let b=c-a

Using Pythagorean, the length of each of the two diagonal pieces will be $(b^2+0.25)^{0.5}$



The total length of the fence will be $a + 2\sqrt{b^2 + \frac{1}{4}}$

We're given the area of each section is 1/3. Thus:

1/3 = a/2 + b/4

a = (4-3b)/6

Let's solve for the length of the fence as a function of b.

$f(b) = (4-3b)/6 + 2\sqrt{b^2 + \frac{1}{4}}$

f'(b) $= \frac{-1}{2} + \frac{2b}{\sqrt{b^2 + \frac{1}{4}}} = 0$

4b = $\sqrt{b^2 + \frac{1}{4}}$

$16b^2 = b^2 + \frac{1}{4}$

$15b^2 = \frac{1}{4}$

$b^2 = \frac{1}{60}$

b = $\frac{\sqrt{60}}{60} = \frac{2\sqrt{15}}{60} = \frac{\sqrt{15}}{30}$ =~ 0.129099444874

a = (4-3b)/6 = $\frac{2}{3} - \frac{\sqrt{15}}{60}$ =~ 0.602116944230

Each diagonal piece = $\sqrt{b^2 + \frac{1}{4}} = \sqrt{\frac{15}{900} + \frac{1}{4}} = \sqrt{\frac{240}{900}} = \frac{2\sqrt{15}}{15}$ =~ 0.516397779494

Total fence = $\frac{2}{3} - \frac{\sqrt{15}}{60} + \frac{4\sqrt{15}}{15} = \frac{2}{3} + \frac{15\sqrt{15}}{60} = \frac{2}{3} + \frac{\sqrt{15}}{4}$ =~ 1.634912503219