

**Question:** Solve for x.

$$9^x + 12^x = 16^x$$

**Solution:**

Divide both sides by  $9^x$

$$9^x/9^x + 12^x/9^x = 16^x/9^x$$

$$1 + (12/9)^x = (16/9)^x$$

$$1 + (4/3)^x = ((4/3)^2)^x$$

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$$\text{Let } y=(4/3)^x$$

$$1 + y = y^2$$

Solving for y, using the quadratic equations, gives us the golden ratio...

$$y = (1+\sqrt{5})/2 \approx 1.67272093446233$$

Recall that  $y=(4/3)^x$

$$(4/3)^x = (1+\sqrt{5})/2$$

Take the log of each side:

$$\log((4/3)^x) = \log((1+\sqrt{5})/2)$$

$$x * \log(4/3) = \log((1+\sqrt{5})/2)$$

$$x = \log((1+\sqrt{5})/2) / \log(4/3)$$

$$x = (\log(1+\sqrt{5}) - \log(2)) / (\log(4) - \log(3))$$

$$\approx 1.67272093446233$$