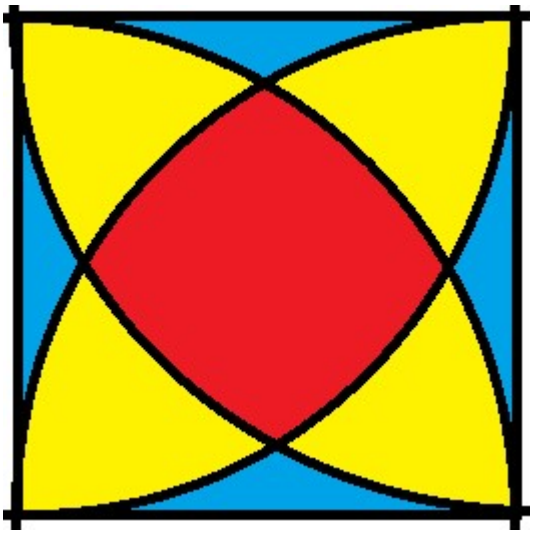
Question:



There is a square of side length 1. Each corner of the square is the center of a circle of radius one. What is the area of the red region, where all four circles overlap?

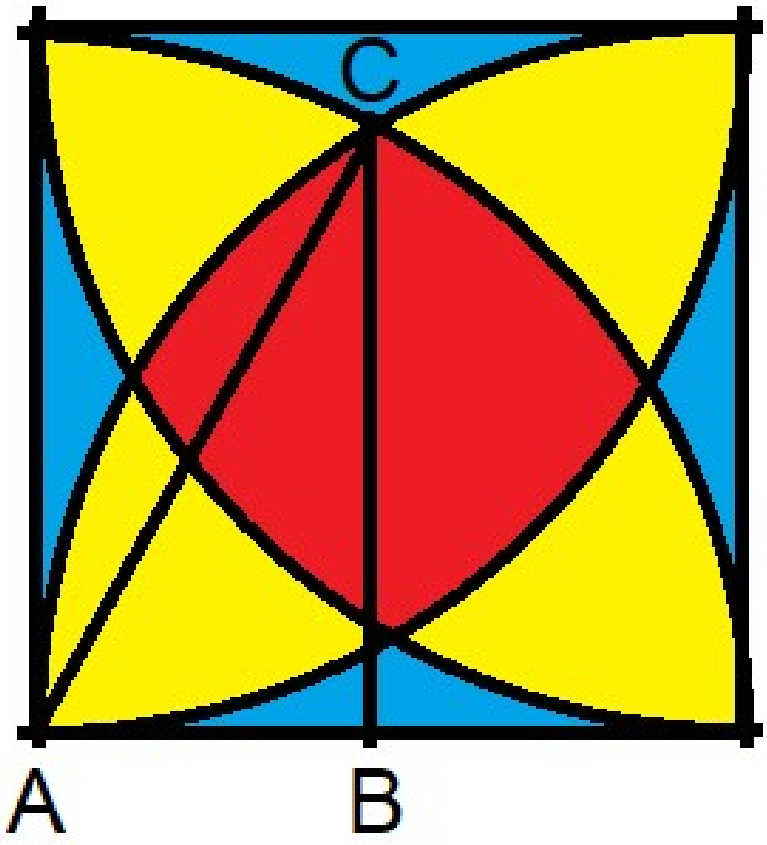
See next page for answer.

Answer:

$1 + \pi/3 - \sqrt{3}$ = apx.
0.315146743627720452626768119587295261122917879314654645602507884650672451853269629128198755023457113651756554089020732519061336441517908265096797623948453390622001468998340591633878394773231643386862306163927058318654928725486294742340423083863353504721515

See next page for solution.

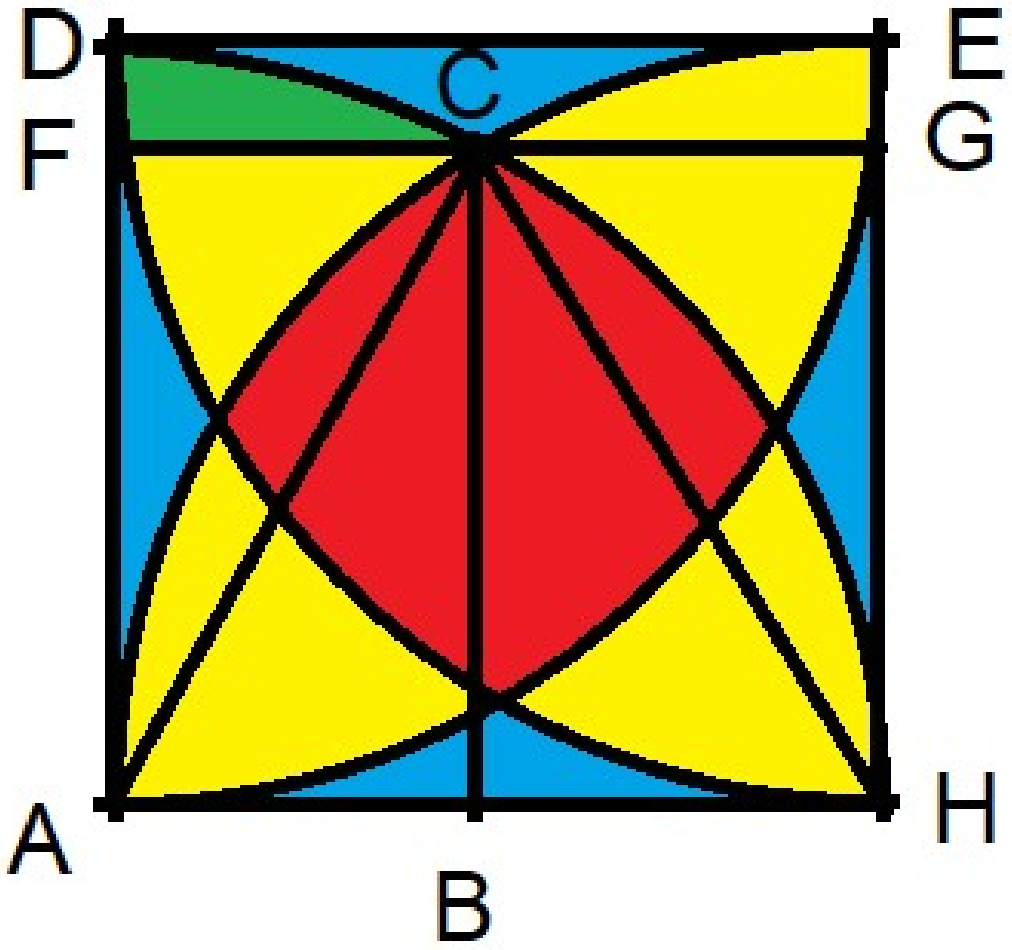
Solution



Let's start by finding the height of triangle ABC. We know AB = 0.5 and AC = 1. So, by the Pythagorean formula, BC = sqrt(3)/2 =~ 0.8660.

For our next step, let's find the area of triangle ABC. It has a base of length 0.5 and a height of sqrt(3)/2. The area of a right triangle is (1/2)*base*height. In that case this is (1/2)*(1/2)*(sqrt(3)/2) = sqrt(3)/8 =~ 0.2165.

Next, let's find the area of green region in the next diagram.



Angle DAC = Angle CAB, because AC and FB are diagonals of the same rectangle. What is angle CAB? That is half of angle ACH, which is 60 degrees, because every side of triangle ACH = 1. So, angle CAB = 30 and thus, does angle FAC.

30 degrees is ½ of the entire 360-degree circle. The entire circle has an area of π. So, slice DAC has an area of $\pi/12 =\sim 0.2618$.

We already established triangle ABC has an area of sqrt(3)/8. This is the same as triangle AFC. Subtracting out triangle FAC from slice DAC, gives us the green region = $\pi/12$ – sqrt(3)/8 = 0.0453.

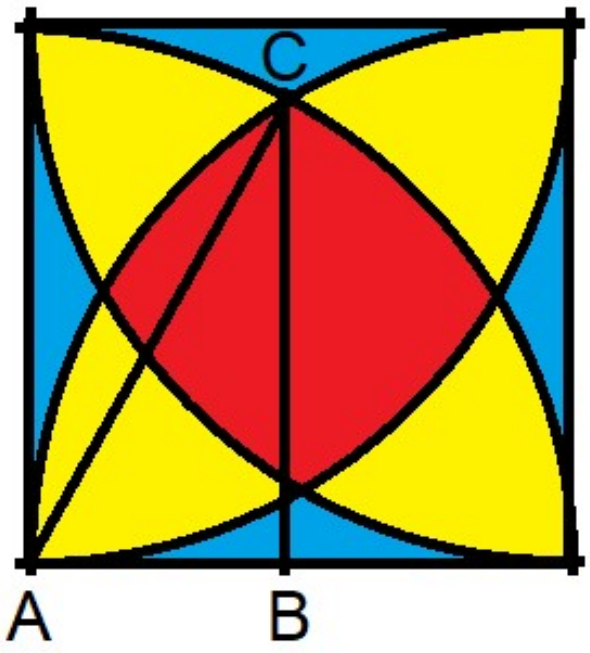
Next, let's find the area of any one blue region. To do that, let's first find the area of rectangle DEFG.

We already know AF=BC = $\sqrt{3}/2$. So, DF = 1 – AF = 1-$\sqrt{3}/2$ = 0.1340. Thus rectangle DEFG = 1*(1-$\sqrt{3}/2$) = 1- $\sqrt{3}/2$.

A blue region is going to be rectangle DEFG minus two green regions:

Blue = (1-$\sqrt{3}/2$) – 2*($\pi/12$ – $\sqrt{3}/8$) = 1 - $\sqrt{3}/2$ – $\pi/6$ + $\sqrt{3}/4$ = 1 - $\sqrt{3}/4$ – $\pi/6$ = 0.0434.

Next, let's find the area of any one yellow region. Let's revert back to the original diagram.



The area of two blue regions plus one yellow region equal the area of the square minus ¼ the area of any one circle = 1 – $\pi/4$ = 0.2146.

We already know the area of a blue region is 1 - $\sqrt{3}$ /4 – $\pi/6$. So we have:

$1 - \pi/4 = \text{yellow} + 2*\text{blue}$

$1 - \pi/4 = \text{yellow} + 2*(1 - \sqrt{3}/4 - \pi/6)$

$\text{Yellow} = 1 - \pi/4 - 2*(1 - \sqrt{3}/4 - \pi/6)$

$= 1 - \pi/4 - 2 + \sqrt{3}/2 + \pi/3$

$= -1 + \pi/12 + \sqrt{3}/2 = 0.1278$

The region is the entire square minus four yellows and four blues:

$\text{Red} = 1 - 4*(-1 + \pi/12 + \sqrt{3}/2) - 4*(1 - \sqrt{3}/4 - \pi/6)$

$= 1 + 4 - \pi/3 - 2*\sqrt{3} - 4 + \sqrt{3} + 2\pi/3$

$= 1 + \pi/3 - \sqrt{3} = 0.3151.$