

Question

Two dice are rolled until either event happens:

A) A total of 2 is rolled.

B) A total of 7 is rolled twice consecutively.

What is the probability the total of 2 occurs first?

Answer

$7/13 = \text{apx. } 53.85\%$

Scroll down for the solution.

Solution

The probability of rolling a total of 12 is $(1/6)*(1/6) = 1/36$

The probability of rolling a total of 7 is $1/6$ (I hope I don't need to explain why. If I do, this problem is too hard for you).

Let x be the probability of the total of 2 happening first from the starting point or after any total other 2 and 7, which resets the state to the starting state.

Let y be the probability of a total of 2 happening first after a roll of 7.

Let $\text{pr}(z)$ = Probability of rolling a total of z .

We can express x as follow:

$$x = \text{pr}(12)*1 + \text{pr}(7)*y + (1-\text{pr}(7)-\text{pr}(12))*x$$

$$x = 1/36 + (1/6)*y + (29/36)*x$$

$$(7/36)x = (1/36)+(1/6)*y$$

$$(1) \quad 7x = 1 + 6y$$

We can express y as follows:

$$y = \text{pr}(12)*1 + \text{pr}(7)*0 + (1-\text{pr}(7)-\text{pr}(12))*x$$

$$y = (1/36)*1 + (1/6)*0 + (29/36)x$$

$$(2) \quad y = (1 + 29x)/36$$

Next, substitute the value of y in equation (2) into equation (1).

$$7x = 1 + 6 \cdot (1+29x)/36$$

$$252x = 36 + 6 \cdot (1+29x)$$

$$252x = 36 + 6 + 174x$$

$$78x = 42$$

$$x = 42/78 = 21/39 = 7/13$$

Not that it was asked, but $y = 6/13$