Question: Two six-sided dice are rolled until a total of 11 is achieved 18 consecutive times in a row. What is the expected number of rolls this will take?

Answer: 41,660,902,667,961,039,785,742

Solution:

Let x = the number of rolls required.

Let p = probability of rolling a total of 11 in one throw.

Let pr(z) = probability of event z.

There are two ways to roll a total of 11 out of the 36 possible outcomes of two dice, so the probability of rolling a total of 11 is 2/36 = 1/18. So p = 1/18.

We can express x as follows.

$x = (1-p)*(1+x) + p*(1-p)*(2+x) + p^2*(1-p)*(2+x) + p^3*(1-p)*(3+x) + \ldots + p^{17}*(1-p)*(18+x) + \ldots + p^{18}*18$.

Let's rearrange terms…

$x = (1+x) + p*(x+2-x-1) + p^2*(x+3-x-2) + p^3*(x+4-x-3) + \ldots + p^{17}*(x+18-x-17) + p^{18}*(18-x-18)$

$x = (1+x) + p*1 + p^2*1 + p^3*1 + \ldots + p^{17}*1 + p^{18}*(-x)$

$x = 1+x + p + p^2 + p^3 + \ldots + p^{17} - p^{18}*x$

$p^{18}x = \sum_{i=0}^{17} p^i$

$$p^{18}x = \sum_{i=0}^{\infty} p^i - \sum_{i=18}^{\infty} p^i$$

$$p^{18}x = \sum_{i=0}^{\infty} p^i - p^{18} \sum_{i=0}^{\infty} p^i$$

$$p^{18}x = (1-p^{18}) \sum_{i=0}^{\infty} p^i$$

$$p^{18}x = (1-p^{18}) * \frac{1}{1-p}$$

$$p^{18}x = \frac{1-p^{18}}{1-p}$$

$$x = \frac{p^{-18}-1}{1-p}$$

Let's divide the numerator and denominator by p:

$$x = \frac{p^{-19}-(\frac{1}{p})}{(\frac{1}{p})-1}$$

$$x = \frac{p^{-19}-\left(\frac{1}{p}\right)+\left(\frac{1}{p}-1\right)-\left(\frac{1}{p}-1\right)}{(\frac{1}{p})-1}$$

$$x = \frac{p^{-19}-\left(\frac{1}{p}\right)+\left(\frac{1}{p}-1\right)}{(\frac{1}{p})-1} - 1$$

$$x = \frac{p^{-19}-1}{(\frac{1}{p})-1} - 1$$

Recall that p = 1/18. Replacing that for p, we get:

$$x = \frac{18^{19} - 1}{18 - 1} - 1$$

x = 41,660,902,667,961,039,785,742

If we assume a constant global population of 8 billion and everybody rolls dice at a rate of once a second 24 hours a day and 365 days a year, then it would take, on average, 165132 years for such event to occur.

The general formula for the expected number of trials required for an event of probability p to happen n times in a row is

$$\frac{(\frac{1}{p})^{n+1} - 1}{\frac{1}{p} - 1} - 1$$