

Question: A swimming pool is full of saltwater. A hose lets fresh water into the pool at a constant rate. As fresh water pours in, water flows out at the other end at the same rate. After an amount of water equal to the volume of the pool has poured into the pool, what ratio of saltwater will remain?

Answer: $\frac{1}{e} = 36.7879\%$.

Solution:

Let:

s = ratio of salt water still in pool.

t = time.

c = constant of integration.

Since we're not given the size of the pool, let's say it is 1000 cubic meters. We're also not given a flow rate. Let's say that is 1 cubic meter per second

The rate at which saltwater leaves the pool at any given time t can be expressed as:

$$ds/dt = -s/1000$$

$$ds = -s/1000 dt$$

$$-1000/s ds = dt$$

Integrate both sides:

$$-1000 \ln(s) = t + c$$

We're given that at $t=0$, $s = 1$

$$-1000 \ln(1) = 0 + c$$

$$c = 0$$

So, we can forget about the constant of integration. The relationship between time and saltwater content is:

$$-1000 \ln(s) = t$$

The question asks what is the value of s after 1,000 cubic meters of fresh water has circulated through the pool.

$$-1000 \ln(s) = 1000$$

$$\ln(s) = -1$$

$$s = e^{-1} = \frac{1}{e} = 36.7879\%.$$