

Question: A swimming pool is full of saltwater. The pool holds 1000 cubic meters of water. A hose lets fresh water into the pool at a rate of one cubic meter per hour. As fresh water pours in, water flows out at the other end at the same rate. How long does it take for the pool to contain only 1% salt water?

Answer: $2000 \ln(10) \approx 4605.17$ hours.

Solution:

Let:

s = ratio of salt water still in pool.

t = time.

c = constant of integration.

The rate at which saltwater leaves the pool at any given time t can be expressed as:

$$ds/dt = -s/1000$$

$$ds = -s/1000 dt$$

$$-1000/s ds = dt$$

Integrate both sides:

$$-1000 \ln(s) = t + c$$

We're given that at $t=0$, $s = 1$

$$-1000\ln(1) = 0 + c$$

$$c = 0$$

So, we can forget about the constant of integration. The relationship between time and saltwater content is:

$$-1000 \ln(s) = t$$

The question asks when will $s = 0.01$?

$$-1000 \ln(s) = t$$

$$-1000 \ln(0.01) = t$$

$$-1000 (\ln(1) - \ln(100)) = t$$

$$-1000(0 - \ln(100)) = t$$

$$-1000(-\ln(10^2)) = t$$

$$2000 \ln(10) = t$$

$$t = 2000 \ln(10) \approx 4605.170186 \text{ hours.}$$