Volume of Gabriel's Horn.

$\int_1^\infty \pi(\frac{1}{x})^2 dx =$

$$\pi \int_1^\infty x^{-2} dx =$$

- π ($\frac{1}{x}$ for x from 1 to ∞) =

π

Surface Area of Gabriel's Horn.

y = $\frac{1}{x}$

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

Surface area = $\int_1^\infty \pi \ \frac{2}{x} \sqrt{1 + (\frac{dy}{dx})^2} dx =$

$2\pi \int_1^\infty \ \frac{1}{x} \sqrt{1 + (-\frac{1}{x^2})^2} dx =$

$$2\pi \int_1^{\infty} \frac{1}{x} \sqrt{1 + \frac{1}{x^4}}\, dx =$$

Let's put aside the $\sqrt{1 + \frac{1}{x^4}}$ term for the moment, noting the expression will always be greater than 1 for any 1<= x <=∞.

$$2\pi \int_1^{\infty} \frac{1}{x}\, dx =$$

2π (ln(x) for x = 1 to ∞) =

2π (ln(∞) – ln(1)) =

2π (∞ – 0) =

∞

If this equals ∞, then adding $\sqrt{1 + \frac{1}{x^4}}$ to the integral, which, again, is always equal or greater to 1, will still equal ∞.