Question

There is a rubber band that is one meter long. An ant is at one end of it. The ant travels to the other end at a speed of 1 centimeter per second. Starting from the time the ant starts moving, the rubber band expands at a rate of 1 meter per second. How long does it take for the ant to reach the other end?

Answer

e^{100} – 1 seconds =~
26,881,171,418,161,400,000,000,000,000,000,000,000,000 seconds.

Solution

Recall that the derivative of distance covered is speed.

The speed of the ant, relative to the full length of the rubber band, at time t is:

$$\text{speed(t)} = \frac{1}{100(1+t)}$$

Integrating that speed function will give us the distance covered. We need to solve for the time it will take the distance covered, relative to the total length of the rubber band, to be 1. Let's call that time x.

$$\int_0^x \frac{1}{100(1+t)}\, dt = 1$$

Let's integrate!

$\frac{\ln(1+t)}{100}$ from 0 to x = 1

ln(1+x) – ln(1+0) = 100

ln(1+x) = 100

1+x = e^{100}

x = e^{100} - 1

Note: I've seen this question asked where everything is the same except the rubber band starts at 1km long and expands that much per second. With 100,000 centimeters in a kilometer, the answer in that case would be e^{100000} - 1