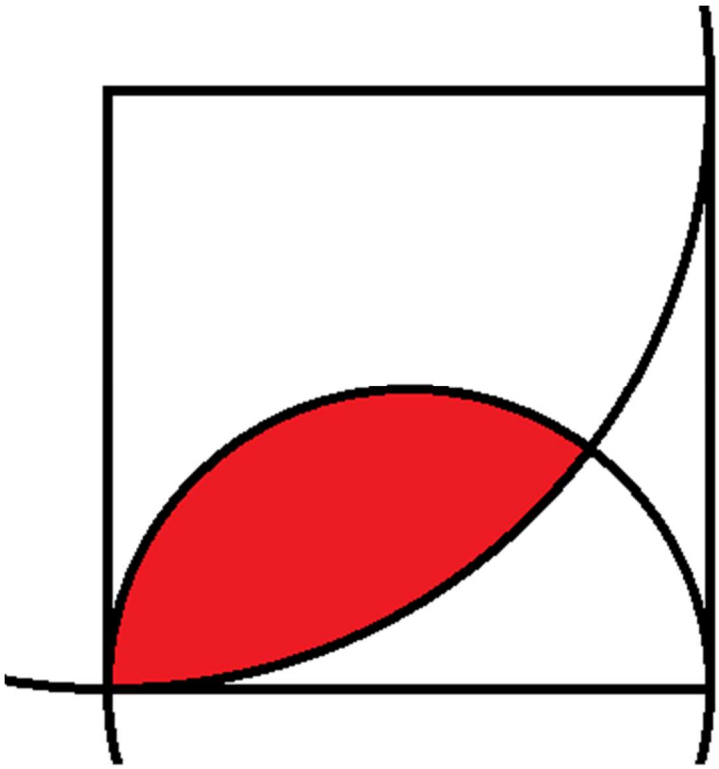
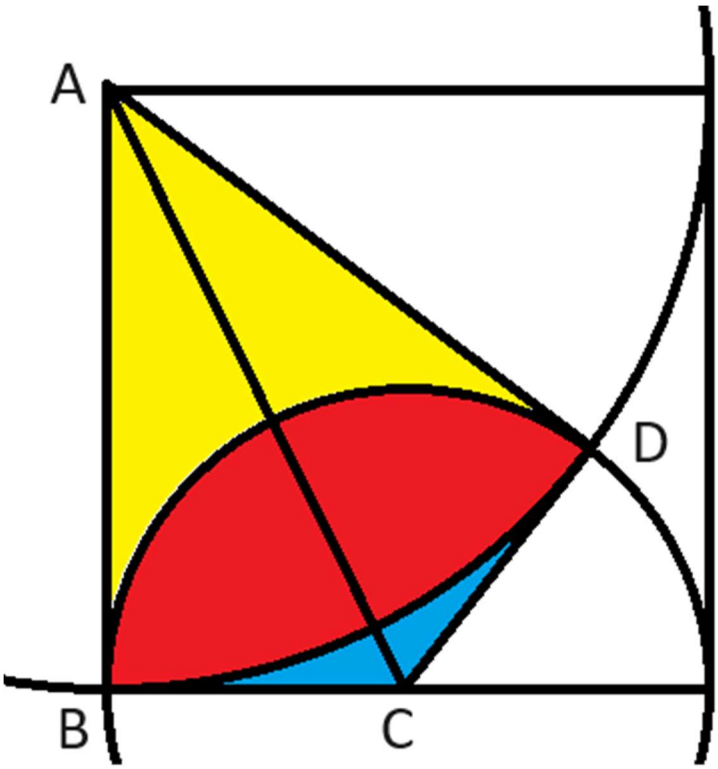
Q: The square above has side length 2. One arc is of a circle of radius 1 and the other radius 2. What is the area of the red region? It is allowed to express your answer in trig functions.



A: $4*(\arctan(2) + 4*\arctan(1/2) - 2) =\sim 3.84695661518926$

S: Consider the following diagram:



Since AB=4 and AB and AD are both the radii of the circle with center A and AB=4, then AD=4.

Since BC=2 and BC and CD are both the radii of the circle with center C and BC=2, then CD=2

AC is the hypotenuse of triangles ABC and ACD.

We have shown that triangles ABC and ACD both have sides of the same length, thus must be the same size triangle.

The area of triangle ABC = base*height/2 = 2*4/2 = 4.

Since ABC and ACD are equivalent triangles, ACD must also have area 4.

The area of the kite consisting of the yellow + red + blue regions is the sum of the areas of triangles ABC and ACD = 4+4 = 8.

Angle BAC = Angle CAD = arctan(2/4) = arctan(1/2). Thus, angle BAD = 2arctan(1/2).

The area of the whole circle with center A is $\pi * 4^2$.

The ratio of the whole circle centered at A to the slice consisting of the yellow and red sections is

$2\arctan(1/2)/(2\pi) = \arctan(1/2)/\pi$

The area of the yellow + red slice is $\pi * 4^2 * \arctan(1/2)/\pi = 16*\arctan(1/2)$.

To review:

Yellow + red + blue = 8

Yellow + red = 16*arctan(1/2)

Thus, blue = 8 - 16*arctan(1/2)

Next, consider the red + blue area.

Angle BCA = angle DCA = tan(2).

The area of the whole circle with center C is $\pi * 2^2$.

The ratio of the whole circle centered at C to the slice consisting of the blue and red sections is

$2\arctan(2)/(2\pi) = \arctan(2)/\pi$

The area of the blue + red slice is $\pi * 2^2 * \arctan(2)/\pi = 4*\arctan(2)$.

To review:

Blue = 8 - 16*arctan(1/2).

Red + blue = 4*arctan(2).

Thus, red = 4*arctan(2) – (8 - 16*arctan(1/2))

= 4*(arctan(2) – 2 + 4*arctan(1/2)) =~ 3.84695661518926.

This problem taken from the YouTube channel “Mind Your Decisions.”

Link: https://www.youtube.com/watch?v=6YvlHt8dlHQ