Phantom bonuses

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1 Introduction

The game is defined by a list of payouts u_1, u_2,..., u_ℓ, and a list of probabilities p_1, p_2, ..., p_ℓ, $\sum_{i=1}^{\ell} p_i = 1$. We allow u_i to be rational numbers, not just integers, to include games like blackjack, n-play video poker or the banker bet in baccarat. We assume that the casino has the advantage, so $\sum_{i=1}^{\ell} p_i u_i < 0$.

The player can bet any positive integer k up to his current bankroll or the maximum bet b, whichever is smaller, and his bankroll increases by $u_i k$ with probability p_i. To cater for blackjack or poker type games, only the player's initial bet is limited, and we allow the player to borrow money for splits, doubles or raises if necessary, but he has to stop if he loses and ends up with a negative bankroll. Each game is independent of all others.

The player has a phantom bonus m without wagering requirements. At any point he can decide to cash in, if his bankroll is greater than m, then m is deducted from his balance in and the bonus is lost, if his bankroll is m or less, he forfeits his whole balance.

2 The question

If the player's current bankroll is n, what strategy should he follow to maximize the expectation of the real money he can cash in after deducting the bonus, and what is this expectation a_n?

n can be a rational number whose denominator is the least common multiple of the denominators of the u_i, for example, when dealing with blackjack,

n can be a half integer, but the stake is always an integer and let us also define $a_n = n$ for $n < 0$. Define the value of the phantom bonus to be $a_n - \max(0, n-m)$, this is the amount the player expects to gain by playing optimally instead of cashing in immediately.

In games involving an element of skill, we assume that the player is playing a fixed strategy, possible adjustments to playing strategy in view of the changed expected values are not considered, strategy will only mean betting strategy.

3 The solution

In theory, there is an easy way to calculate to a_n. Start with $a_{n,0} = n$ for $n < 0$, $a_{n,0} = 0$ for $0 \leq n \leq m$, and $a_{n,0} = n - m$ for $n > m$. Then for each n, calculate $\sum_{i=1}^{\ell} p_i a_{n+u_i k,0}$ for all integers k, $1 \leq k \leq \min(n, b)$, and let $a_{n,1}$ be the maximum of these values and $a_{n,0}$. We are calculating the optimal strategy using $a_{n,0}$ as approximation to a_n. Repeat with $a_{n,1}$, and so on. In general, let

$$a_{n,j+1} = \max\Big(a_{n,j}, \Big\{\sum_{i=1}^{\ell} p_i a_{n+u_i k,j} | 1 \leq k \leq \min(n, b)\Big\}\Big).$$

$a_{n,j} \leq n$ for all j, since the casino has the advantage, so the expected value of the player's cash-in cannot exceed current bankroll, even ignoring the bonus. $a_{n,j} \leq a_{n,j+1}$, and it is a well-known theorem in analysis that a bounded monotonically increasing sequence has a limit, so $a_n = \lim_{j\to\infty} a_{n,j}$ exists. Unfortunately, this procedure as described involves an infinite amount of calculation.

I conjecture that the phantom bonus is worthless, i.e., $a_n = n - m$ if

$$n \geq N = \frac{m \sum_{u_i<0} p_i u_i}{\sum_{i=1}^{\ell} p_i u_i}, \tag{1}$$

and that if the player plays a game in which all the payouts are integers ≥ -1 and $b \geq N$, then the optimal strategy is simply to bet everything if $n < N$ and to cash in if $n \geq N$. Under these conditions N is exactly the point at which the expected value of betting everything is the same as that of cashing in. If the game only has two possible outcomes, $u_1 = -1$ with probability $1-p$ and $u_2 = u$, a positive integer, with probability p, then a_n can be calculated by finding the integer r such that $N/(u+1)^r \le n < N/(u+1)^{r-1}$, and then $a_n = p^r((u+1)^r n - m)$, so a_n is a piecewise linear function on n. I cannot prove all this, but I have quite good numerical evidence. The idea for the proof I have depends on some kind of convexity of the sequence a_n. (Convexity means $a_n \le (a_{n-1} + a_{n+1})/2$, in other words, that the player should always take a bet with no house edge. I would only need something weaker, that a_n/n increases with n, and $a_{n+k} \le a_n + k$, the latter meaning that value of the phantom bonus decreases as n increases.)

If the conjecture is true, then the calculations can be restricted to $n < N$, which makes each iterative step finite.

Another approach to the problem is that if the optimal strategy is known for each n, then we can write down the linear equations relating the a_n and solve them, this gives exact answers, unfortunately the optimal strategy is not known in advance.

The method described below is a combination of iterations and linear equations.

Step 1. Calculate N by (1). Set $a_n = n$ for $n < 0$, $a_n = 0$ for $0 \le n \le m$, and $a_n = n - m$ for $m < n \le N$.
Step 2. Check for each n, $0 < n \le N$, whether betting the maximum possible amount improves the current value of a_n, and if so, update the value of a_n. Do this 10, 20 or 50 times, the exact number does not seem to make much difference in the running time. For some reason I do not understand, going in decreasing order of n gives faster convergence than in increasing order.
Step 3. Calculate the optimal strategy using the current values for a_n, and solve the resulting linear equations. This is equivalent to using the currently optimal strategy and letting it converge. This is the slowest step because it involves solving a large, but sparse system of linear equations, usually consisting of several hundred equations.
Step 4. For each n, calculate the expected value of betting any of possible bet sizes using the current values of a_n. If this does not give an improvement for any n, then the current numbers are the exact values of a_n, and we can also deduce the optimal strategy. If improvement is possible, then go back to Step 2.

The calculations were carried out using *Mathematica*, which can handle rational numbers with arbitrary numerators and denominators, so it can give exact answers. These exact answers can be used to justify the use of the unproven conjecture retrospectively. This method produced results reasonably quickly for all the games are I looked at.

4 The results

The tables in the Appendix show the results for various games. The size of the phantom bonus was always taken to be $m = 10$. The first column shows the maximum bet, the second column the largest bankroll at which it is still in the player's interest to play, rather than to cash in. The remaining columns show the value of the phantom bonus, $a_n - \max(0, n-m)$, from $n = 5$ to 40 in steps of 5. These numbers were rounded to 3 decimal places. The probabilities for blackjack were taken from a simulation by Michael Shackleford, the "Wizard of Odds", available on the web at http://www.wizardofodds.com/games/blackjack/bjapx4.html.

I believe that the problem scales linearly as long as the payouts are integers, so choosing $m = 10$ is not a real restriction.

The tables confirm some of the expected phenomena. The value of the phantom bonus, and also the size of the bankroll at which the player should stop playing both increase with b. The value of the phantom bonus is maximal at $n = m$. The house edge is not the primary factor in determining the value of the phantom bonus, a large probability of losing and a small probability of winning a large amount is the good for the player. Among all the roulette bets, which all have the same house edge, betting on a single number is the best. Similarly, Jacks or Better video poker with doubling is better than without doubling, while the house edge is the same. The line bet (6 numbers) in roulette with house edge 2.7% is better for the player than coin flip with probability of winning 0.495 and house edge 1%. The numbers for coin flip with probability of winning 0.499 and house edge 0.2% are similar to those for Jacks or Better video poker with house edge 0.46%.

5 Details of the strategy

Assume that all the payouts are integers ≥ -1. Empirical evidence suggests and I may be able to prove it rigorously, too, that if player's bankroll is a multiple of b, he should always bet the maximum, b. In this case the numbers a_{jb} ($j = 0, 1, 2, \ldots$) satisfy the recurrence relation $a_{jb} = \sum_{i=1}^{\ell} p_i a_{(j+u_i)b}$. Let $u = \max\{u_i | 1 \le i \le \ell\}$. The solution is of the form $a_{jb} = \sum_{i=0}^{\ell} c_i \lambda_i^j$, where the λ_i, $i = 0, 1, \ldots, u$, are the roots of the characteristic equation $1 = \sum_{i=1}^{\ell} p_i \lambda^{u_i}$, assuming that the roots are distinct.

Let kb be the highest multiple of b at which the player should still play. The coefficients c_i, $i = 0, 1, \ldots, u$, can be determined from the equations $a_0 = 0$, $a_{(k+1)b} = (k+1)b - m$, $a_{(k+2)b} = (k+2)b - m, \ldots$, $a_{(k+u)b} = (k+u)b - m$.

Define $\alpha_{jb,t}$ by $\alpha_{jb,t} = \sum_{i=1}^{\ell} p_i \alpha_{(j+u_i)b,t}$ and $\alpha_{0,t} = 0$, $\alpha_{(t+1)b,t} = (t+1)b - m$, $\alpha_{(t+2)b,t} = (t+2)b - m, \ldots$, $\alpha_{(t+u)b,t} = (t+u)b - m$. These are the values of the a_{jb} using the assumption that $k = t$. k can be found as the largest value of t for which $\alpha_{tb,t} \ge tb - m$, but unfortunately, there is no exact method of solving for k.

Example 1. Coin flip with the probability of winning p

The characteristic equation is $1 = (1-p)\lambda^{-1} + p\lambda$, the roots are $\lambda_0 = 1$, $\lambda_1 = (1-p)/p$, and the solution is

$$\alpha_{jb,t} = \frac{(((1-p)/p)^j - 1)(tb - m)}{((1-p)/p)^{t+1} - 1}.$$

Now let $m = b = 10$, $p = 0.495$. The table below shows the value of $\alpha_{tb,t}$ calculated using the above formula for various values of t.

t	1	2	3	4	5	6	7	8	9	10
$tb - m$	0	10	20	30	40	50	60	70	80	90
$\alpha_{tb,t}$	4.95	13.2	22.3	31.7	41.2	50.9	60.6	70.4	80.2	89.97

For $t = 9$, $\alpha_{tb,t} > tb - m$, for $t = 10$, $\alpha_{tb} < tb - m$, so $k = 9$. This means that the player should still play with a bankroll of 90, but not with a bankroll of 100, which agrees with the table in the appendix, which says that largest bankroll at which the player should still play is 98.

Example 2. Betting on a single number in roulette

The characteristic equation is $1 = 36\lambda^{-1}/37 + \lambda^{35}/37$, it cannot be solved exactly, but *Mathematica* is able to provide numerical solutions. Let $m = 10$, $b = 50$, the results are in the table below.

t	1	2	3	4	5	6	7
$tb - m$	40	90	140	190	240	290	340
$\alpha_{tb,t}$	48.4	96.8	145.3	193.8	242.3	290.9	339.5

For $t = 6$, $\alpha_{tb} > tb - m$, for $t = 7$, $\alpha_{tb,t} < tb - m$, so $k = 6$. This means that the player should still play with a bankroll of 300, but not with a bankroll of 350, which agrees with the table in the appendix, which says that largest bankroll at which the player should still play is 332.

There are some curious phenomena that I do not understand. The table below shows some values of n and the corresponding optimal bet size when betting on a single number in roulette with $m = b = 10$. If the last digit of n is 7, 8 or 9, then it is never correct to bet 10, but only 7, 8, or 9, respectively. There are many other numbers for which betting the maximum is not correct, in all of these cases the correct bet size is the last digit of n.

n	1	2	3	4	5	6	7	8	9	10
bet	1	2	3	4	5	6	7	8	9	10
n	11	12	13	14	15	16	17	18	19	20
bet	10	10	10	10	10	10	7	8	9	10
n	101	102	103	104	105	106	107	108	109	110
bet	10	10	10	10	10	6	7	8	9	10
n	141	142	143	144	145	146	147	148	149	150
bet	10	10	10	10	5	6	7	8	9	10
n	171	172	173	174	175	176	177	178	179	180
bet	10	10	10	4	5	6	7	8	9	10
n	201	202	203	204	205	206	207	208	209	210
bet	10	10	3	4	5	6	7	8	9	10
n	221	222	223	224	225	226	227	228	229	230
bet	10	2	3	4	5	6	7	8	9	10
n	241	242	243	244	245	246	247	248	249	250
bet	1	2	3	4	5	6	7	8	9	10

If we consider the strategy for the split bet with the same m and b, then this phenomenon does not start until much later, the player should bet the maximum possible for all $n \leq 157$.

I can only guess at the reasons. It seems that multiples are b are somehow preferable, so when the optimal bet is not b, it is the difference between n and the largest multiple of b less than n, so that if the player loses, his bankroll will be a multiple of b and from that point on he will always bet b. This phenomenon also occurs in other games especially near the upper bound. The reason why 7, 8 and 9 are different in the first example seems to be that the maximum bankroll at which the player should still play in the first example is 252, and winning bet with a stake of 7 or more would get him to this bound or above it. In the second example, the maximum bankroll at which the player should still play is 198, this cannot be reached by betting 10 or less, this is why the optimal strategy behaves differently. Other examples also support this, but I have no rigorous explanation for this observation, I can only speculate.

6 The effects of using the wrong strategy

The traditional wisdom about phantom bonuses is that the player should bet big, but the previous section shows that betting the maximum is not always correct. I also calculated the expectations if the player only bet $\min(n, b)$ and the difference from the a_n was very small, typically only a few thousandths or even less, so for practical purposes always betting the maximum possible is a good strategy.

I also considered what happens if the player chooses a different target, for example he gets a 100% match bonus on his deposit and aims to increase his bankroll tenfold, so if $m = 10$ as in the previous calculations, he aims for 200. The typical situation is that if the optimal strategy suggests that he should aim higher, say, for 400, he does not gives up much in terms of expectation by stopping at 200. On the other hand, if the optimal strategy suggests stopping earlier, say, at 100, then trying to go for 200 can be expensive.

7 Appendix

Baccarat (8 decks): player bet

1	30	2.308	4.953	2.984	1.46	0.443	0.01		
2	40	2.916	6.046	4.388	2.976	1.807	0.919	0.312	0.025
5	61	3.573	7.245	6.018	4.896	3.881	2.976	2.185	1.51
10	84	3.921	7.95	6.98	6.119	5.263	4.515	3.774	3.143
20	114	4.16	8.436	7.716	7.105	6.396	5.784	5.179	4.684
50	169	4.339	8.797	8.294	7.838	7.475	6.956	6.533	6.17
100	225	4.42	8.962	8.532	8.172	7.873	7.438	7.131	6.846
200	286	4.46	9.044	8.66	8.339	8.071	7.698	7.398	7.185
500	371	4.47	9.065	8.712	8.38	8.071	7.804	7.536	7.269

Baccarat (8 decks): tie bet

1	24	1.99	4.39	2.288	0.796				
2	30	2.598	5.534	3.664	2.211	1.035	0.154		
5	42	3.33	6.82	5.453	4.216	3.097	2.084	1.168	0.339
10	50	3.411	7.612	6.175	5.452	4.215	3.497	2.447	1.729
20	57	3.496	7.612	6.894	6.177	4.871	4.153	3.435	2.717
50	61	3.577	7.612	6.894	6.177	5.459	4.741	4.023	3.305
100	63	3.577	7.612	6.894	6.177	5.459	4.741	4.023	3.305

Blackjack

1	57.5	3.32	6.884	5.587	4.418	3.377	2.467	1.692	1.057
2	80	3.801	7.736	6.726	5.786	4.911	4.104	3.365	2.696
5	123	4.154	8.423	7.722	7.053	6.414	5.806	5.228	4.679
10	171	4.353	8.811	8.29	7.808	7.315	6.86	6.401	5.973
20	236	4.491	9.077	8.677	8.32	7.936	7.592	7.25	6.944
50	355	4.545	9.308	8.984	8.759	8.489	8.244	7.985	7.784
100	475	4.603	9.422	9.151	8.977	8.754	8.568	8.354	8.201

Coin flip, probability of winning 0.45

1	14	0.448	1.669						
2	17	1.016	2.717	0.502					
5	23	1.93	4.288	2.171	0.694				
10	30	2.436	5.413	3.508	2.03	0.929	0.116		
20	39	2.734	6.075	4.604	3.5	2.254	1.341	0.532	
50	52	2.87	6.379	5.125	4.175	3.225	2.5	2.	1.5
100	54	2.87	6.379	5.125	4.175	3.225	2.5	2.	1.5

Coin flip, probability of winning 0.48

1	19	1.244	3.099	0.868					
2	25	1.929	4.301	2.181	0.719	0.02			
5	36	2.787	5.806	4.076	2.619	1.457	0.615	0.12	
10	48	3.251	6.773	5.32	4.111	2.979	2.06	1.275	0.671
20	64	3.562	7.42	6.311	5.458	4.355	3.564	2.795	2.204
50	91	3.747	7.807	6.995	6.265	5.736	4.989	4.404	3.885
100	115	3.822	7.963	7.165	6.589	6.012	5.344	4.952	4.56
200	129	3.822	7.963	7.209	6.589	6.012	5.436	4.952	4.56

Coin flip, probability of winning 0.49

1	25	1.936	4.302	2.191	0.719	0.03			
2	34	2.589	5.459	3.624	2.127	0.996	0.271		
5	51	3.323	6.781	5.38	4.127	3.026	2.084	1.308	0.704
10	69	3.713	7.578	6.453	5.465	4.51	3.674	2.895	2.218
20	93	3.983	8.129	7.281	6.589	5.76	5.063	4.385	3.856
50	137	4.17	8.511	7.917	7.369	6.958	6.362	5.882	5.446
100	179	4.25	8.674	8.168	7.703	7.353	6.874	6.483	6.128
200	228	4.291	8.757	8.259	7.871	7.483	7.059	6.765	6.471
500	254	4.291	8.757	8.276	7.871	7.483	7.095	6.765	6.471

Coin flip, probability of winning 0.495

1	34	2.594	5.46	3.628	2.129	0.998	0.274		
2	47	3.159	6.487	4.979	3.656	2.516	1.58	0.846	0.336
5	71	3.753	7.582	6.489	5.474	4.54	3.687	2.919	2.236
10	98	4.065	8.212	7.362	6.589	5.827	5.136	4.462	3.856
20	134	4.282	8.651	8.022	7.476	6.854	6.308	5.765	5.305
50	203	4.449	8.988	8.554	8.157	7.819	7.382	7.021	6.68
100	270	4.524	9.14	8.791	8.465	8.214	7.861	7.57	7.302
200	354	4.566	9.225	8.92	8.636	8.415	8.122	7.876	7.649
500	502	4.588	9.268	8.975	8.723	8.478	8.233	8.02	7.823

Coin flip, probability of winning 0.499

1	73	3.753	7.582	6.489	5.474	4.54	3.688	2.92	2.237
2	102	4.085	8.213	7.381	6.592	5.844	5.14	4.478	3.86
5	159	4.403	8.823	8.261	7.717	7.191	6.682	6.192	5.72
10	222	4.566	9.15	8.735	8.337	7.94	7.561	7.183	6.822
20	309	4.681	9.381	9.081	8.799	8.5	8.218	7.937	7.674
50	478	4.777	9.574	9.377	9.186	9.013	8.811	8.627	8.449
100	660	4.825	9.669	9.52	9.376	9.251	9.097	8.961	8.83
200	900	4.856	9.731	9.613	9.501	9.405	9.284	9.178	9.079
500	1335	4.879	9.777	9.689	9.594	9.508	9.436	9.349	9.266

European roulette: straight up (single number)

1	105	4.183	8.398	7.646	6.928	6.243	5.592	4.975	4.391
2	140	4.395	8.83	8.26	7.73	7.197	6.701	6.204	5.744
5	201	4.611	9.229	8.852	8.483	8.119	7.764	7.419	7.084
10	252	4.654	9.459	9.123	8.934	8.606	8.422	8.104	7.924
20	295	4.688	9.459	9.324	9.189	8.885	8.671	8.535	8.4
50	332	4.711	9.459	9.324	9.189	9.054	8.919	8.784	8.649
100	347	4.721	9.459	9.324	9.189	9.054	8.919	8.784	8.649
200	355	4.726	9.459	9.324	9.189	9.054	8.919	8.784	8.649
500	359	4.726	9.459	9.324	9.189	9.054	8.919	8.784	8.649

European roulette: split bet (2 numbers)

1	76	3.868	7.798	6.792	5.85	4.974	4.165	3.423	2.75
2	103	4.161	8.377	7.606	6.89	6.188	5.54	4.909	4.329
5	152	4.459	8.932	8.42	7.923	7.44	6.972	6.518	6.078
10	198	4.547	9.218	8.781	8.462	8.041	7.734	7.336	7.046
20	249	4.619	9.304	9.054	8.919	8.559	8.261	8.031	7.896
50	302	4.68	9.382	9.102	8.919	8.784	8.649	8.514	8.378
100	327	4.704	9.418	9.14	8.919	8.784	8.649	8.514	8.378
200	341	4.704	9.438	9.159	8.919	8.784	8.649	8.514	8.378
500	349	4.704	9.438	9.171	8.919	8.784	8.649	8.514	8.378

European roulette: street bet (3 numbers)

1	63	3.63	7.352	6.166	5.077	4.085	3.194	2.406	1.724
2	85	3.981	8.032	7.113	6.265	5.45	4.704	3.997	3.356
5	126	4.337	8.695	8.074	7.475	6.898	6.343	5.811	5.302
10	167	4.457	9.054	8.536	8.151	7.658	7.292	6.823	6.475
20	216	4.562	9.192	8.875	8.649	8.246	7.906	7.614	7.407
50	277	4.638	9.311	9.018	8.736	8.514	8.378	8.243	8.108
100	309	4.668	9.358	9.092	8.804	8.537	8.378	8.243	8.108
200	328	4.668	9.401	9.134	8.843	8.576	8.378	8.243	8.108
500	339	4.668	9.401	9.134	8.868	8.601	8.378	8.243	8.108

European roulette: corner bet (4 numbers)

1	54	3.431	6.982	5.655	4.455	3.388	2.456	1.665	1.02
2	74	3.826	7.74	6.701	5.749	4.848	4.034	3.277	2.608
5	110	4.231	8.492	7.781	7.1	6.449	5.826	5.233	4.671
10	147	4.377	8.902	8.314	7.865	7.312	6.891	6.372	5.977
20	192	4.492	9.071	8.72	8.419	7.946	7.557	7.236	6.968
50	256	4.617	9.261	8.932	8.625	8.338	8.108	7.973	7.838
100	293	4.617	9.35	9.011	8.744	8.444	8.178	7.973	7.838
200	315	4.617	9.35	9.083	8.817	8.509	8.242	7.976	7.838
500	329	4.617	9.35	9.083	8.817	8.55	8.283	8.017	7.838

European roulette: line bet (6 numbers)

1	44	3.097	6.368	4.822	3.469	2.32	1.386	0.68	0.214
2	60	3.564	7.246	6.011	4.897	3.875	2.976	2.179	1.508
5	90	4.048	8.14	7.276	6.458	5.686	4.96	4.281	3.649
10	121	4.248	8.637	7.935	7.366	6.718	6.193	5.599	5.119
20	160	4.388	8.88	8.429	8.069	7.517	7.058	6.651	6.327
50	221	4.503	9.129	8.752	8.39	8.1	7.769	7.484	7.297
100	263	4.539	9.204	8.937	8.563	8.255	7.988	7.675	7.401
200	292	4.563	9.204	8.937	8.671	8.404	8.137	7.815	7.526
500	309	4.563	9.204	8.937	8.671	8.404	8.137	7.871	7.604

European roulette: 9 numbers

1	35	2.68	5.614	3.826	2.344	1.195	0.411	0.027	
2	48	3.226	6.617	5.152	3.862	2.735	1.795	1.037	0.48
5	72	3.805	7.679	6.624	5.641	4.732	3.897	3.139	2.458
10	97	4.068	8.285	7.433	6.722	5.951	5.313	4.623	4.06
20	129	4.275	8.629	8.058	7.574	6.94	6.378	5.888	5.473
50	183	4.389	8.918	8.475	8.042	7.707	7.3	6.976	6.661
100	227	4.462	8.997	8.608	8.342	8.075	7.635	7.254	6.988
200	260	4.462	9.067	8.641	8.342	8.075	7.809	7.542	7.275
500	279	4.462	9.067	8.672	8.342	8.075	7.809	7.542	7.275

European roulette: dozen or column bets (12 numbers)

1	30	2.312	4.96	2.994	1.468	0.448	0.004		
2	40	2.913	6.052	4.393	2.984	1.815	0.925	0.317	0.015
5	60	3.574	7.247	6.021	4.899	3.883	2.977	2.184	1.506
10	81	3.873	7.95	6.941	6.118	5.231	4.512	3.75	3.134
20	108	4.108	8.331	7.668	7.103	6.34	5.687	5.145	4.683
50	155	4.288	8.705	8.22	7.685	7.226	6.84	6.453	6.079
100	200	4.333	8.87	8.36	7.942	7.615	7.348	6.979	6.54
200	233	4.37	8.87	8.475	8.081	7.64	7.348	7.082	6.815
500	249	4.37	8.87	8.475	8.081	7.686	7.348	7.082	6.815

European roulette: even money bets (18 numbers),
no en prison rule or half of your stake back on 0

1	22	1.635	3.778	1.586	0.265				
2	30	2.311	4.966	3.001	1.473	0.457	0.0001		
5	44	3.102	6.377	4.833	3.482	2.333	1.398	0.689	0.219
10	59	3.526	7.249	5.989	4.9	3.866	2.977	2.181	1.502
20	80	3.812	7.836	6.89	6.108	5.193	4.442	3.713	3.111
50	114	4.011	8.245	7.535	6.948	6.487	5.767	5.267	4.837
100	146	4.087	8.402	7.784	7.27	6.876	6.278	5.807	5.5
200	189	4.109	8.447	7.883	7.364	6.876	6.481	6.087	5.692

Full pay Jacks or Better video poker

1	108	4.28	8.577	7.892	7.226	6.579	5.953	5.35	4.769
2	140	4.426	8.867	8.318	7.785	7.262	6.756	6.26	5.783
5	201	4.595	9.197	8.805	8.42	8.043	7.672	7.308	6.952
10	264	4.683	9.393	9.084	8.802	8.501	8.227	7.935	7.669
20	344	4.746	9.516	9.293	9.092	8.847	8.626	8.411	8.219
50	480	4.799	9.622	9.456	9.294	9.148	8.988	8.84	8.695
100	605	4.825	9.675	9.534	9.398	9.274	9.149	9.028	8.904

Full pay Jacks or Better video poker with doubling once on every win

1	149	4.45	8.912	8.385	7.871	7.369	6.88	6.404	5.941
2	196	4.576	9.163	8.754	8.356	7.962	7.578	7.2	6.833
5	284	4.71	9.423	9.141	8.861	8.586	8.314	8.046	7.782
10	374	4.774	9.568	9.346	9.144	8.926	8.727	8.513	8.319
20	485	4.819	9.655	9.497	9.35	9.174	9.015	8.861	8.719
50	680	4.859	9.733	9.614	9.499	9.397	9.279	9.175	9.075
100	860	4.878	9.772	9.673	9.576	9.49	9.392	9.303	9.217