Question: What is the expected number of trials before the sum of random numbers drawn from a uniform distribution from 0 to 1 exceeds a total of 1?

Step 1: Let's first answer the question, "What is the probability that the sum of **two** such random numbers is less than 1?" That answer can be expressed as:

$$\iint_0^{1-x} 1\, dy\, dx =$$

$\int_0^1 1 - x\; dx$ =

$x - \frac{x^2}{2}$ from 0 to 1 =

1- 0.5 = 1/2

Step 2: Let's next answer the question, "What is the probability that the sum of **three** such random numbers is less than 1?" That answer can be expressed as:

$$\iiint_0^{1-x-y} 1\, dz\, dy\, dx =$$

$$\iint_0^{1-x} 1 - x - y\; dy\, dx =$$

$\int_0^1 y - xy - \frac{y^2}{2}$ from $1 - \mathrm{x}$ to 0 dx =

$\int_0^1 0.5 - x + \frac{x^2}{2}\;\; dx$ =

$\frac{x^1}{2} - \frac{x^2}{2} + \frac{x^3}{2}$ from 0 to 1 $=$ 1/6

Step 3: Let's next answer the question, "What is the probability that the sum of **four** such random numbers is less than 1?" Let's change the variables to a-d, because there isn't a letter that comes after z. That answer can be expressed as:

$$\int \;\; \iiint_0^{1-a-b-c} 1\, dd\, dc\, db\, da =$$

$$\iiint_0^{1-a-b} 1 - a - b - c \; dc \; db \; da =$$

$$\iint_0^{1-a-b} \frac{1}{2} - a - b + ab + \frac{a^2}{2} + \frac{b^2}{2} \; db \; da =$$

$$\int_0^1 \frac{b}{2} - \mathrm{ab} - \frac{b^2}{2} + \frac{ab^2}{2} + \frac{ba^2}{2} + \frac{b^3}{6} \text{ from 0 to } 1 - \mathrm{a} \; dx =$$

$$\frac{1}{2} \times \int_0^1 \frac{1}{3} - a + \frac{a^2}{2} - \frac{a^3}{3} \; dx =$$

$$\frac{1}{2} \times \left(\frac{a^1}{3} - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{12} \text{ from 0 to 1}\right) = 1 / 24$$

Step 4: We start to see a pattern forming.

Pr(Sum of 2 numbers from [0,1] < 1) = 1/2!
Pr(Sum of 3 numbers from [0,1] < 1) = 1/3!
Pr(Sum of 4 numbers from [0,1] < 1) = 1/4!

I think it is safe to assume that Pr(Sum of n numbers from [0,1]) = 1/n!

Step 5:

We can see that Pr(exactly n number from [0,1] needed to exceed 1) =
(1 - 1/n!) - (1 - 1/(n-1)!) =
(n / n!) - (1/n!) =
(n-1)/n!

Step 6:

The expected number of numbers from [0,1] needed to exceed 1 =
2 × (1/2!) + 3 × (2/3!) + 4 × (3/4!) + 5 × (4/5!) + ... =
1/1! + 2/2! + 3/3! + 4/4! + ... =
0! + 1! + 1/2! + 1/3! + ... =
e = 2.7182818...